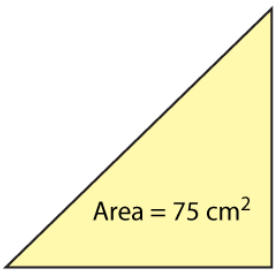
Level 3 Diploma
in
Installing Electrotechnical Systems & Equipment

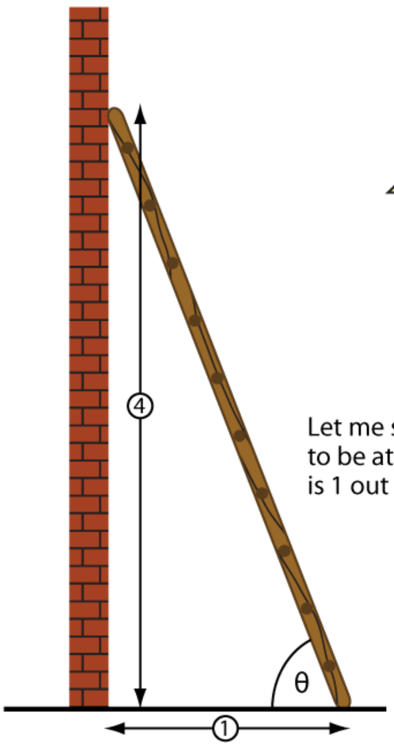
C&G 2357

Unit 309-1 Understand the mathematical principles which are appropriate to electrical installation, maintenance and design work



$$If\ A = \frac{1}{2} b \times h \text{ then } b = \frac{2 \times A}{h}$$

$$so\ A = \frac{2 \times 75}{10} = 15\ cm^2$$



Let me see, ladder wants to be at a ratio of 1:4, that is 1 out for every 4 up.

So, using trigonometry;

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{4}{1} = 4$$

$$\text{so } \theta = \tan^{-1} 4 \approx 76^\circ$$

B&B Training Associates Ltd

Learning Materials for the Electrical Industry

Produced by

B&B Training Associates Ltd

For further information contact:

Mrs A Bratley

23 St Pauls Drive,

Tickton, Nr Beverley,

East Yorkshire

HU17 9RN

Tel:-01964 – 543137

Fax:-01964 – 544109

Email:- sales@bbta.co.uk

timbenstead@bbta.co.uk

terry@bbta.co.uk

Version 1-2011

Aims and objectives

By the end of this study book you will have had:

- the opportunity to identify and apply appropriate mathematical principles which are relevant to electrotechnical work tasks.

Range:

- Fractions and percentages
- Algebra
- Indices
- Powers of 10
- Transposition
- Triangles and trigonometry
- Statistics.

1: Powers of 10

In this session the student will:

- Gain an understanding of mathematical symbols.
- Gain an understanding of number and powers of 10 and the basic principles of engineering standard form?

Introduction

Numbers are a strange concept. They define size, quantity, direction and so many other things that we can forget what it is that we are dealing with. You cannot do any form of engineering without a reasonable level of mathematical knowledge.

Mathematics need not be as daunting as many people find it and is simply a way of describing a problem and providing the tools to solve it. For example, choosing a cable size requires the parameters to be described, such as the type of cable used, the current demand of the circuit, the temperature of the surroundings and whether it is grouped with other cables and so on. The calculation comes only after all the data has been gathered to describe the problem.

Symbol	English equivalent
+	Add; add to; plus; is positive; sum
-	Subtract; take away; minus; is negative
×	Times; multiply; product
÷	Divide; quotient
=	Equals; is exactly equal to; the same as
≠	Is not equal to; is not the same as; is different to
≈	Is approximately equal to; is approximately the same as
≤	Is greater than or equal to
≥	Is less than or equal to
()	Brackets – Do what's in the brackets first
∴	Therefore
<	Greater than
>	Less than
±	Plus or minus – often used in an answer
∝	Is proportional to
∞	Infinity
∠	An angle of

As with any language, mathematics uses a language all of its own. The table below describes some of the symbols used:

Powers of 10 and standard form

Numbers used in engineering can often be either very small or very large. Currents flow in the order of millionths of an amp, such as 0.00000001 amps or in the thousands of amps, such as 20000 amps. Energy can be transferred in the order of trillions of joules, such as 5000000000000 joules. You can see that the numbers are either too small or too large to be easy to use. Missing a zero out can have an enormous impact on the solution to a problem.

It is for this reason why we start to use prefixes (letters before the numbers) to give us multiples of a thousand. Most of the common engineering multiples can be seen over the page.

Increasing value

Powers of Ten	Value	Name	Symbol	
1×10^{12}	1 000 000 000 000	Tera	T	
1×10^{9}	1 000 000 000	Giga	G	
1×10^{6}	1 000 000	Mega	M	
1×10^{3}	1 000	kilo	k	
1×10^{2}	100	hecto	h	(rarely used)
1×10^{1}	10	deca	da	(rarely used)
1×10^{0}	1	Unity	—	
1×10^{-1}	0.1	deci	d	(used with the bel to give decibel)
1×10^{-2}	0.01	centi	c	
1×10^{-3}	0.001	milli	m	
1×10^{-6}	0.000001	micro	µ	
1×10^{-9}	0.000000001	nano	n	
1×10^{-12}	0.000000000001	pico	p	

Decreasing value

There are more prefixes than these shown, but these are the most commonly used in engineering.

The common thing is that they are all based on the ***power 10***. This is usually written as:

$$m \times 10^{n}$$

Where: m is any number between 1 and 9 inclusive

n is any power

A number written with one digit (**one and only one digit**) to the left of the decimal point, multiplied by ten which is then raised to some power is said to be written in ***standard form.***

So:

i. $5837 = 5.837 \times 10^{3}$

ii. $0.0415 = 4.15 \times 10^{-2}$

iii. $39400 = 3.94 \times 10^{4}$

Notice that only one digit sits to the left of the decimal point. The number of decimal points moved is the number used for the index[1]. If the decimal point is moved towards the right then we have a negative number, and if we move it to the left, we have a positive index.

Rather than sticking to using numbers only, prefixes can be used. These letters represent a set multiple of 10. For example, 10^{3} can be replaced by the letter ‘k’, and ‘k’ represents a 1000, which is the same as 10^{3}.

i. $5837 = 5.837 \times 10^{3} = 5.837k$

ii. $0.0415 = 4.15 \times 10^{-2} = 41.5 \times 10^{-3} = 41.5m$

iii. $39400 = 3.94 \times 10^{4} = 39.4 \times 10^{3} = 39.4k$

Notice how easy it is to change the decimal point and to add the power or to use the prefix.

Don’t get confused between standard form and the prefixes and numbers that are used in engineering. With standard form, we have only one number in front of the decimal point. With engineering form, we often have more than one number in front of the decimal point.

In each example show as many ways of displaying the number as possible.		
22kW	**340mm**	**220μΩ**
22×10^{3}W	340×10^{-3}m	$220 \times 10^{-6}\Omega$
22000W	0.34m	0.00022Ω

Remember that each of the prefixes, the letters, have a meaning.

[1] The index is the power that the number (in this case 10) is raised to.

Exercise 1.

Express each number in standard form and as a prefix.

1) i) 2200000 Ω
   ii) 10450 m
   iii) 0.020 A
   iv) 0.0000345 A
   v) 567000 W.

Express the following as whole numbers.

2) i) 0.37 kW
   ii) 8 μF
   iii) 55 TWh
   iv) 220 kΩ
   v) 0.5 mH.

Don't get confused between the letters used for the unit and the letters that relate to the number itself.

3) Convert the following to engineering form to include both powers of ten and the prefix letter.
   i) 12467238
   ii) 0.00045612
   iii) 213
   iv) 34673
   v) 44668822561849
   vi) 0.000000000005
   vii) 0.00000000341
   viii) 12613790
   ix) 0.0000045

2: Basic operations

In this session the student will:

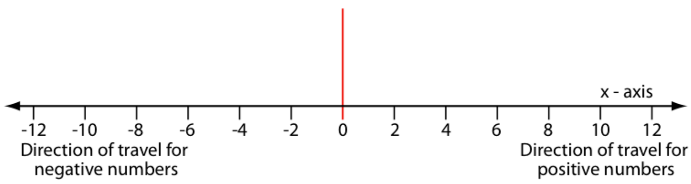
- Gain an understanding of directed numbers.
- Practice basic operator rules including BODMAS.

Most of this section will require you to recognise that Maths is a language like any other, and that it is a means of communicating.

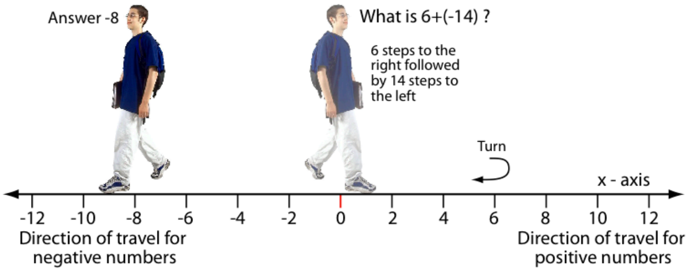
Directed numbers

Many people find it strange when they see negative numbers and start thinking that an answer having a minus sign stuck in front of it must be wrong. This would be wrong. There are many things in life where a negative number makes absolute sense, for example, you spend too much money and your bank account goes into the red.

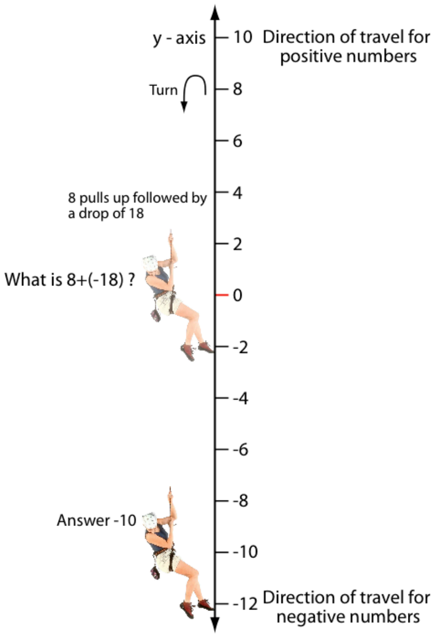
In engineering there are many occasions where it is necessary to express negative numbers. These will include, a.c. waveforms, direction of current flow and the like. Consider a line.



Numbers extending to the right of zero are considered positive, whilst numbers to the left of zero are considered negative. However, if a person were to walk, say, 6 m in one direction then turn and walk 14 m in the opposite direction, he/she would be at a distance of -8 m from where they started, or 8 m from where they started in the opposite direction to where they originally walked.



Similarly, we get the same idea when climbing.



A climber starts at a certain point and moves upward (or in a positive direction) by 8 m. The climber then descends 18 m (or in a negative direction). The overall height he/she has climbed is 10 m downwards (or negative).

Negative numbers, therefore, make sense when there is a recognised starting point against which we can reference our values. The recognised reference point is zero.

1. If no sign is present at the front of a number then we assume that it is positive. Therefore, in this instance the three is positive even though the sign is not seen, as in the second example shown.

$$3+4=7$$
$$+3+4=7$$

2. If ***unlike signs*** are together in a calculation when adding or subtracting, the overall sign depends upon which number is the largest.

$$3+(-4)=3-4=-1$$
$$4-(+6)=4-6=-2$$

3. Where ***like signs*** are together in a calculation when adding or subtracting, the overall sign is ***positive***.

$$3-(-4)=3+4=7$$
$$4+(+6)=4+6=10$$

4. Where the numbers have ***unlike signs*** when multiplying or dividing, the overall sign is ***negative***.

$$\frac{4}{-3}=-\frac{4}{3}$$
$$3\times(-4)=-12$$

5. Where the numbers have ***like signs*** when multiplying or dividing, the overall sign is ***positive***.

$$\frac{-4}{-3}=\frac{4}{3}$$
$$-3\times-4=12$$

6. In arithmetic operations, the order in which the operations are performed is critical:

    i) To determine the values of operations contained in brackets

    ii) Percentages/ratios i.e. ***"of"***

    iii) Division

    iv) Multiplication

    iv) Addition

    v) Subtraction.

    This can be shortened to an acronym, **BODMAS**.

Brackets

Of

Division

Multiplication

Addition

Subtraction.

The basic laws governing the use of brackets are as follows:

1. The use of brackets when adding does not affect the result.

$$2+(3+4)=(2+3)+4=2+3+4$$

2. The use of brackets when multiplying does not affect the result.

$$2\times(3\times4)=(2\times3)\times4=2\times3\times4$$

3. A number placed outside of a bracket requires that the contents of the bracket must be multiplied by that number.

$$2(3+4)=(2\times3)+(2\times4)=6+8=14$$

or

$$2(3+4)=2(7)=14$$

4. Adjacent brackets indicate multiplication.

$$(2+3)(3+4)=(2\times3)+(2\times4)+(3\times3)+(3\times4)=6+8+9+12=35$$

or

$$(2+3)(3+4)=(5)\times(7)=35$$

5. When an expression contains inner and outer brackets, the inner brackets should be dealt with first.

$$2(2-3(3-5))$$
$$2(2-3(-2))$$
$$2(2+6)=2(8)=16$$

Exercise 2.

1)	$7(4+2)$	2)	$5+2\times7$	3)	$7\times4+2$
4)	$4\times3-2+4$	5)	$9+6\div3$	6)	$2\times14-(3+9)$
7)	$(9+6)\div3$	8)	$3\times7-2\times8$	9)	$(7-2)3$
10)	$2+3+6\times4$	11)	$(-7+2)6$	12)	$4((2+3)5+6)$
13)	$7\div2(4(3+6))$	14)	$7\times2(4(3+6))$	15)	$4((2-3)5+6)$
16)	$(2+3+6)4$	17)	$22\div2+8\times2$		

18) $(24-14)-(5-2(34+4))$

19) $4(-4+6)\div(4+(-2))$

20) $\dfrac{3(4-7)}{(2+3)(4-5)}$

3: Fractions

In this session the student will:

- Gain an understanding of adding, subtracting, multiplying and dividing fractions.

Fractions are often approached with dread by a large number of people. This is a shame, as the following of some basic steps will allow even those with little confidence to achieve sufficient levels of proficiency for this and many other courses.

Probably the best way of gaining the necessary skills is to work through a number of examples.

Adding

1. Simplify $\frac{1}{3}+\frac{3}{8}$

The top number in any fraction is always called the ***numerator*** and the bottom number is always called the ***denominator***. Therefore, in this instance, the ***numerators*** are 1 and 3, and the ***denominators*** are 3 and 8.

Step 1

Find the ***LCM*** (Lowest Common Multiple) of the two denominators. That is find a number that both denominators can be divided into: this is often called the ***Lowest Common Denominator***.

In this case the LCM is $3 \times 8 = 24$

All you have to do is to multiply the two denominators; that is the bottom two numbers together!

What we have now is $\frac{\quad}{24}$

Step 2

Divide the denominator of the first fraction into the LCM.

The LCM is 24: which is then divided by the 3 from the fraction $1/3$.

$$\frac{1}{3} \to\to 24 \div 3 \to\to 8$$

Multiply the 8, gained by our division, by the numerator of the fraction; which in this case is 1.

$$8 \times 1 \to\to \frac{8+}{24}$$

Notice that the addition sign can be put in straight away.

Step 3

Repeat this for the second fraction. Here, the denominator is 8. However, you still divide the lowest common multiple by the 8. This gives us an answer of 3.

$$\frac{3}{8} \to\to 24 \div 8 \to\to 3$$

Then multiply the 3 by the numerator of the fraction.

$$3 \times 3 \to\to \frac{8+9}{24}$$

Step 4

What we have now is the hard work done and the full working out would look like this:

$$\frac{1}{3} + \frac{3}{8} = \frac{(8 \times 1) + (3 \times 3)}{24} = \frac{8+9}{24} = \frac{17}{24}$$

2. Simplify $\frac{3}{4}+\frac{5}{8}$

Step 1

Find the LCM. In this instance the two numbers don't need multiplying together as both the '4' and the '8' are multiples of one another. That is the 8 can be divided by the 4. This means that rather than multiplying the 4 and the 8 together, and getting a LCM of 32, we can just use the 8.

$$\frac{\quad}{8}$$

This is the lowest common multiple. Now for the rest of the process, and remember this is exactly what it is, a process. It can be followed every time.

Step 2

Divide the denominator of the first fraction into the LCM.

Multiply the 2, gained by our division, by the numerator of the fraction; which in this case is 3.

$$2\times 3 \rightarrow\rightarrow \frac{6+}{8}$$

Step 3

Repeat this for the second fraction. Here, the denominator is 8. However, you still divide the lowest common multiple by the 8. This gives us an answer of 1.

$$\frac{5}{8}\rightarrow\rightarrow 8\div 8\rightarrow\rightarrow 1$$

Then multiply the 1 by the numerator of the fraction.

$$1\times 5\rightarrow\rightarrow \frac{6+5}{8}$$

Step 4

What we have now is the hard work done and the full working out would look like this:

$$\frac{3}{4}+\frac{5}{8}=\frac{(2\times 3)+(1\times 5)}{8}=\frac{6+5}{8}=\frac{11}{8}=1\frac{3}{8}$$

Dealing with improper or top-heavy fractions can seem to be difficult. In our answer we would have been correct to state that the answer is $\frac{11}{8}$. However, it is more normal not to express an answer in an improper form.

Quite simply $\frac{11}{8} = 8\overline{)11} = 8\overline{)11}^{\,1^{r3}} = 1\frac{3}{8}$

- The numerator (11) is divided by the denominator (8) and the correct quantity of whole numbers is determined (1).
- The remainder becomes the new numerator (3).
- The denominator stays the same (8).
- The whole number (1) becomes the standalone number.

The process is always the same.

Subtraction

This is very similar to working with addition of fractions. All that you have to do is remember to subtract rather than add; everything else is the same.

1. Simplify $\frac{2}{3}-\frac{1}{6}$

Step 1

Find LCM

The lowest common multiple is a number that both the denominators will divide into. Although 3 and 6 do divide into 18 (3×6) they also divide into 12 and 6. Therefore, the LCM becomes 6.

Step 2

Divide the denominator of the first fraction into the LCM.

The LCM is 6: which is then divided by the 3 from the fraction $^2/_3$.

$$\frac{2}{3}\to\to6\div3\to\to2$$

Multiply the 2, gained by our division, by the numerator of the fraction; which in this case is 2.

$$2\times2\to\to\frac{4-}{6}$$

Notice that the minus sign can be put in straight away.

Step 3

Repeat this for the second fraction. Here, the denominator is 6. This gives us an answer of 1.

$$\frac{1}{6}\to\to6\div6\to\to1$$

Then multiply the 1 by the numerator of the fraction.

$$1\times1\to\to\frac{4-1}{6}$$

Step 4

What we have now is the hard work done and the full working out would look like this:

$$\frac{2}{3}-\frac{1}{6}=\frac{(2\times 2)-(1\times 1)}{6}=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$$

Make a careful note that although it is always useful to simplify things it is not always necessary. In this instance, it is merely preferable.

To simplify the last example we need to find something that is common to both the numerator and denominator. Well, the numerator is 3 and the denominator is 6, so the common value is 3 because it can divide into both 6 and 3.

$$\frac{\cancel{3}^{1}}{\cancel{6}^{2}}=\frac{1}{2}$$

2. Simplify $\frac{3}{4}-\frac{5}{8}$

$$\frac{3}{4}-\frac{5}{8}=\frac{(2\times 3)-(1\times 5)}{8}=\frac{6-5}{8}=\frac{1}{8}$$

Improper fractions

Whilst we are still within this area, we will need to consider what is known as ***improper fractions***. These are fractions that mix whole numbers and fractions of numbers. They are not much more difficult but do require a little more care. Follow the example below.

1. Simplify $3\frac{2}{3}+2\frac{1}{6}$

There are two possible ways of doing this; the most common way is to convert the whole numbers to fractions and then to do the sum as shown in the previous two examples.

Step 1

Convert the two fractions into improper fractions.

$$\text{Since } 1=\frac{3}{3} \quad 3=3\times\frac{3}{3}=\frac{9}{3}$$

$$\text{Then } 3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$$

Similarly

$$\text{Since } 1=\frac{6}{6} \quad 2=2\times\frac{6}{6}=\frac{12}{6}$$

$$\text{Then } 2\frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}$$

What we have now is a basic addition of fractions.

Step 2

$$3\frac{2}{3}+2\frac{1}{6}=\frac{11}{3}+\frac{13}{6}$$

Step 3

Find the lowest common denominator and sort out the numerators. The LCM is 6 as both 3 and 6 divide into 6.

$$\frac{11}{3}+\frac{13}{6}=\frac{(2\times11)+(1\times13)}{6}=\frac{22+13}{6}=\frac{35}{6}=5\frac{5}{6}$$

The simplifying may seem complex but it isn't really.

$$\frac{35}{6}=6\overline{)35}=6\overline{)35}^{\,5^{r5}}=5\frac{5}{6}$$

2. $3\frac{2}{3}-2\frac{1}{6}$

Again, you should remember to follow the process. Everything is carried out in the same way as before.

Convert the two fractions into improper fractions.

$$\text{Since } 1=\frac{3}{3} \quad 3=3\times\frac{3}{3}=\frac{9}{3}$$
$$\text{Then } 3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$$

Similarly

$$\text{Since } 1=\frac{6}{6} \quad 2=2\times\frac{6}{6}=\frac{12}{6}$$
$$\text{Then } 2\frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}$$

What we have now is a basic subtraction of fractions.

Step 2

$$3\frac{2}{3}-2\frac{1}{6}=\frac{11}{3}-\frac{13}{6}$$

Step 3

Find the lowest common denominator and sort out the numerators. The LCM is 6 as both 3 and 6 divide into 6.

$$\frac{11}{3}-\frac{13}{6}=\frac{(2\times11)-(1\times13)}{6}=\frac{22-13}{6}=\frac{9}{6}=1\frac{1}{2}$$

Multiplication

This is probably the most straightforward way of working with fractions.

1. Simplify $\frac{3}{7}\times\frac{14}{15}$

This type of fraction is probably the most straightforward to solve. You have to multiply the top line together, and then multiply the bottom line together.

$$\frac{3}{7}\times\frac{14}{15}=\frac{3\times14}{7\times15}=\frac{42}{105}$$

2. Simplify $\frac{2}{3}\times\frac{1}{5}\times2$

This is no more complicated than the first example. Remember to multiply the top line and then the bottom line. The whole number acts as if it were on the top line.

$$\frac{2}{3}\times\frac{1}{5}\times2=\frac{2\times1\times2}{3\times5}=\frac{4}{15}$$

Division

Again, this is not a difficult operation to come to terms with. It is very similar to multiplication but with one little twist.

1. Simplify $\frac{3}{7}\div\frac{12}{21}$

The best way of doing this type of problem is to invert (turn upside down) the second fraction and change the sign, then to follow the same procedure as for multiplication.

$$\frac{3}{7}\div\frac{12}{21}=\frac{3}{7}\times\frac{21}{12}=\frac{\cancel{63}^{3}}{\cancel{84}^{4}}=\frac{3}{4}$$

You can see now how this problem has become less complicated. You now just have to follow the same technique as if you were multiplying a fraction.

2. Simplify $\frac{2}{5} \div \frac{4}{7}$

Remember to follow the process.

$$\frac{2}{5} \div \frac{4}{7} = \frac{2}{5} \times \frac{7}{4} = \frac{\cancel{14}^{7}}{\cancel{20}_{10}} = \frac{7}{10}$$

Notice that the process is the same.

You should also remember that the application of BODMAS still applies.

Exercise 3.

1) $\frac{1}{2} + \frac{2}{5}$

2) $\frac{2}{9} + \frac{1}{7}$

3) $\frac{1}{7} + \frac{2}{3}$

4) $\frac{2}{9} - \frac{1}{7}$

5) $\frac{2}{9} - \frac{1}{7} + \frac{2}{3}$

6) $10\frac{3}{7} - 8\frac{2}{3}$

7) $5\frac{3}{13} + 3\frac{3}{4}$

8) $\frac{3}{4} \times \frac{5}{9}$

9) $\frac{13}{17} \times 4\frac{7}{11} \times 3\frac{4}{19}$

10) $\frac{3}{4} \div \frac{5}{9}$

11) $\frac{3}{4} \div 1\frac{4}{5}$

12) $\frac{3}{8} \div \frac{45}{64}$

13) $\frac{1}{2} + \frac{3}{5} \div \frac{9}{15} - \frac{1}{3}$

14) $\left(\frac{2}{3} \times 1\frac{1}{4}\right) \div \left(\frac{2}{3} + \frac{1}{4}\right) + 1\frac{3}{5}$

15) $\dfrac{2\frac{1}{2} - 1\frac{1}{3}}{1\frac{1}{4} + 2\frac{3}{8}}$

16) $\dfrac{2\frac{1}{2} \div 9\frac{1}{6}}{3\frac{1}{4} + 1\frac{1}{12}}$

4: Decimals, ratios and percentages

In this session the student will:

- Gain an understanding of the use of decimals, ratios and percentages.

In this section, we will consider three areas that often lead to basic problems with maths, decimals, ratios and percentages.

Decimals

We have just finished looking at fractions. Where numbers are expressed as fractions then they call them ***vulgar fractions***, for example $\frac{1}{2}$ or $\frac{5}{7}$. Such fractions can also be expressed as ***decimal fractions*** or ***decimals***.

Decimals are based on the number ten (10). This means we can simply create a table and fill in the columns for our decimal.

For example, the number 2459.23 has:

- **Two** thousands
- **Four** hundreds
- **Five** tens
- **Nine** ones
- **Two** tenths
- **Three** hundredths.

1000s	100s	10s	1s	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
2	4	5	9	2	3	0

Here that number is expressed in a table. The decimal point is positioned between the whole numbers; ones and higher, and the fractions, which are parts of a whole number. So $^2/_{10}$ is $^2/_{10}$ of one (1).

Now, the fraction part of that number is the part that exists to the right-hand side of the decimal point. In the first example, it is the highlighted numbers, 2459.**23**.

$$0.23 = \frac{2}{10} + \frac{3}{100}$$
$$\frac{2}{10} = \frac{20}{100}$$
$$\frac{20}{100} + \frac{3}{100} = \frac{23}{100}$$

The number 0.23 is the same as the fraction $\frac{23}{100}$. This cannot be made any simpler by cancelling down, and so our initial number becomes $2459\,^{23}/_{100}$

Have a look at another example. What will be the vulgar fraction of 0.26?

$$0.26 \text{ means } \frac{2}{10} \text{ and } \frac{6}{100}$$
$$\frac{2}{10} = \frac{20}{100} \quad \text{multiply numerator and denominator by 10}$$
$$0.26 = \frac{20}{100} + \frac{6}{100} = \frac{26}{100} \quad \text{cancel down}$$
$$\frac{\cancel{26}^{13}}{\cancel{100}^{50}} = \frac{13}{\underline{\underline{50}}}$$

Therefore, the vulgar fraction of 0.26 is $\frac{26}{100}$.

For the following examples, turn the decimal fractions into vulgar fractions.

1) 123.4
2) 0.156
3) 1765.05
4) 9876754.006
5) 2.5

Working out is shown over the page.

1) 123.4

$$0.4=\frac{\cancel{4}^{2}}{\cancel{10}^{5}}=\underline{\underline{\frac{2}{5}}}$$ divide both numerator and denominator by 2

$$123.4=\underline{\underline{123\frac{2}{5}}}$$

2) 0.156

$$0.156=\frac{\cancel{156}^{39}}{\cancel{1000}^{250}}=\underline{\underline{\frac{39}{250}}}$$ divide both numerator and denominator by 4

$$0.156=\underline{\underline{\frac{39}{250}}}$$

3) 1765.05

$$0.05=\frac{\cancel{5}^{1}}{\cancel{100}^{20}}=\underline{\underline{\frac{1}{20}}}$$ divide both numerator and denominator by 5

$$0.05=\underline{\underline{\frac{1}{20}}}$$

4) 9876754.006

$$0.006=\frac{\cancel{6}^{3}}{\cancel{1000}^{500}}=\underline{\underline{\frac{3}{500}}}$$ divide both numerator and denominator by 2

$$9876754.006=\underline{\underline{9876754\frac{3}{500}}}$$

5) 2.5

$$0.5=\frac{\cancel{5}^{1}}{\cancel{10}^{2}}=\underline{\underline{\frac{1}{2}}}$$ divide both numerator and denominator by 5

$$2.5=\underline{\underline{2\frac{1}{2}}}$$

Remember that the cancelling down is part of the process. Choose the simple things first to cancel.

Exercise 4.

Change the following into vulgar fractions.

1) 0.3 $\left[\dfrac{3}{10}\right]$

2) 0.001 $\left[\dfrac{1}{1000}\right]$

3) 0.5 $\left[\dfrac{1}{2}\right]$

4) 1.05 $\left[1\dfrac{1}{20}\right]$

5) 2.06 $\left[2\dfrac{3}{50}\right]$

6) 0.125 $\left[\dfrac{1}{8}\right]$

7) 0.08 $\left[\dfrac{2}{25}\right]$

8) 1.45 $\left[1\dfrac{9}{20}\right]$

9) 0.4343 $\left[\dfrac{4343}{10000}\right]$

10) 7.07 $\left[7\dfrac{7}{100}\right]$

Ratio and proportion

Quite simply ***ratio*** means ‘compare’. For example, the ratio of 100 m to 400 m means 100 m compared to 400 m. This can then be simplified to express the same ratio in smaller terms and without the units.

100 m compared with 400 m is the same ratio as 1 m compared with 4 m or 1 compared with 4.

This is very ‘wordy’ and as such we like to simplify. So:

100 m compared with 400 m is the same as:

1:4

The use of the colon cuts out all the words.

There are many ways in which ratios can be made and simplified. The following examples all vary slightly, but have the basic principles in mind.

Simplify the following ratios.

1) $32:96$

2) $7:21$

3) $16:24$

4) $9:15:27$

5) $2:8:32$

My answers are over the page.

1)

$32:96$ divide both side by 32

$1:3$

2)

$7:21$ divide both side by 7

$1:3$

3)

$16:24$ divide both side by 8

$2:3$

4)

$9:15:27$ divide all by 3

$3:5:9$

5)

$2:8:32$ divide all by 2

$1:4:16$

It may not be appropriate always to divide to simplify. There will be occasions where it is necessary to multiply, especially when we are dealing with fractions within ratios.

Simplify the following ratios.

1) $2:\frac{1}{2}$

2) $\frac{4}{5}:6$

3) $\frac{2}{3}:\frac{4}{5}$

4) $\frac{1}{4}:\frac{3}{7}$

5) $3:\frac{4}{3}$

The working out is shown over the page.

1)

$2:\frac{1}{2}$ multiply both side by 2

$2\times 2:\frac{2}{2}=4:1$

2)

$\frac{4}{5}:6$ multiply both side by 5

$\frac{4\times 5}{5}:6\times 5$

$\frac{20}{5}:30=4:30$

$\cancel{4}^{2}:\cancel{30}^{15}=2:15$

$\frac{4}{5}:6=2:15$

3)

$\frac{2}{3}:\frac{4}{5}$ multiply both side by the common denominator 15

$\frac{\cancel{30}^{3}}{\cancel{3}^{1}}:\frac{\cancel{60}^{12}}{\cancel{5}^{1}}$ cancel down

$\cancel{3}^{1}:\cancel{12}^{4}$ cancel down

$\frac{2}{3}:\frac{4}{5}=1:4$

4)

$\frac{1}{4}:\frac{3}{7}$ multiply both side by the common denominator 28

$\frac{\cancel{28}^{7}}{\cancel{4}^{1}}:\frac{\cancel{84}^{12}}{\cancel{7}^{1}}$ cancel down

$\frac{1}{4}:\frac{3}{7}=7:12$

5)

$$3:\frac{4}{3} \quad \text{multiply both side by 3}$$

$$3\times 3:\frac{\cancel{12}^{4}}{\cancel{3}^{1}}=9:4$$

Percentages

Percent is a joining together of two words, ***per*** and ***cent*** which simply means '***of a hundred***'.

For example: $45\% = \frac{45}{100} = 0.45$

- To convert a decimal fraction to a percentage we simply need to multiply by 100.
- To convert a vulgar fraction to a percentage we need to convert it to a decimal fraction and then multiply by 100.

We'll consider a number of examples.

1) 0.67
2) 0.25
3) 1.45
4) 0.05
5) 12.5.

The working out is shown below.

1) $0.67\times 100 = 67\%$
2) $0.25\times 100 = 25\%$
3) $1.45\times 100 = 145\%$
4) $0.05\times 100 = 5\%$
5) $12.5\times 100 = 1250\%$

The following examples look at vulgar fractions being turned into decimals first.

1) $\frac{3}{4}$

2) $\frac{2}{3}$

3) $\frac{1}{7}$

4) $\frac{4}{5}$

5) $\frac{1}{16}$.

The working out is not really much more complicated, and you can make use of your calculators.

1)

$$\frac{3}{4}=0.75$$
$$0.75\times 100=75\%$$

2)

$$\frac{2}{3}=0.667$$
$$0.667\times 100=66.7\%$$

3)

$$\frac{1}{7}=0.1429$$
$$0.1429\times 100=14.29\%$$

4)

$$\frac{4}{5}=0.8$$
$$0.8\times 100=80\%$$

5)

$$\frac{1}{16} = 0.0625$$
$$0.0625 \times 100 = 6.25\%$$

Complete the table below.

Percentage %	Decimal	Vulgar fraction
24		
	0.15	
		$2\frac{2}{3}$
		$\frac{6}{7}$
	1	
$17\frac{1}{2}$		
	1.10	
		$1\frac{1}{12}$

The worked out table is shown over the page.

Percentage %	Decimal	Vulgar fraction
24	0.24	$\frac{6}{25}$
15	0.15	$\frac{3}{20}$
266.7	2.667	$2\frac{2}{3}$
85.7	0.857	$\frac{6}{7}$
100	1	1
$17\frac{1}{2}\%$	0.175	$\frac{7}{40}$
110	1.10	$1\frac{1}{10}$
108.3	1.083	$1\frac{1}{12}$

The answers are in the shaded boxes. The percentage expressed as a decimal is called the ***per unit*** value. This is used almost as often as percentage in engineering.

In normal usage we need to be able determine the percentage of a given quantity. In many areas of engineering it is essential that use is made of percent values; for example, motor efficiencies and the percentage regulation of transformers all make use of percentage values.

The value of a percentage of a given quantity is given by:

$$\frac{\text{percent}}{100} \times \text{quantity}$$

You should be able to see that we have converted the percentage into a decimal before multiplying by the given quantity. This means that the **per unit** value is being used.

$$\text{per unit} \times \text{quantity}$$

We'll look at some examples.

1) What is 23% of 28000 students?

2) A $50\,\text{mm} \times 50\,\text{mm}$ trunking has a space factor of 45%. How much area is that?

3) A 200kW motor is 90% efficient. How much power is wasted?

4) You give approximately 30% of your salary of £14500 to the taxman. How much do you keep?

1)

$$23\% \text{ of } 28000 = \frac{23}{100} \times 28000$$
$$0.23 \times 28000 = 6440$$

2)

$$50 \times 50 = 2500\text{mm}^2$$
$$45\% \text{ of } 2500 = \frac{45}{100} \times 2500$$
$$0.45 \times 2500 = 1125$$

3)

$$90\% \text{ of } 200 = \frac{90}{100} \times 200$$
$$0.9 \times 200 = 180$$
$$200 - 180 = 20\text{kW} \quad \text{wasted power}$$

4)

$$30\% \text{ of } 14500 = \frac{30}{100} \times 14500$$
$$0.3 \times 14500 = £4350$$
$$14500 - 4350 = £10150 \quad \text{money kept}$$

Try the examples over the page before looking at the next exercise.

1) The voltage drop on a length of cable is permitted to be 5%. The supply voltage is:

   a). 230 V

   b). 400 V.

   What will be the maximum permitted volt drop in each case?

2) A transformer has a secondary voltage of 18 V. When it is fully loaded 2% of the voltage is dropped. How much will the secondary voltage be when it is fully loaded?

3) What is 27% of £24500?

4) What will be the percentage loss in a motor if the input power is 5 kW and the output power is 4.6 kW?

5) A circuit breaker rated at 10 A operates when 50 A flows through it. What is the percentage increase in current?

1)

a). $\frac{\text{percent}}{100} \times \text{quantity}$

$\frac{5}{100} \times 230 = 11.5V$

b). $\frac{5}{100} \times 400 = 20V$

2)

$\frac{\text{percent}}{100} \times \text{quantity}$

$\frac{2}{100} \times 18 = 0.36V$ voltage dropped

$V = 18 - 0.36 = 17.64V$ actual voltage

3)

$\frac{\text{percent}}{100} \times \text{quantity}$

$\frac{27}{100} \times 24500 = £6615$

4)

$\frac{\text{percent}}{100} \times \text{quantity} = \text{new value}$

$\frac{\text{new value}}{\text{quantity}} \times 100 = \%$

$\frac{4.6}{5} \times 100 = 92\%$

Be careful here. All that is done is to rearrange things a little to get the actual percentage.

5)

$$\frac{\text{percent}}{100} \times \text{quantity} = \text{new value}$$

$$\frac{\text{new value}}{\text{quantity}} \times 100 = \%$$

$$\frac{50}{10} \times 100 = 500\%$$

Don't forget that it is all right to have a percentage increase greater than 100%. All this means is that the value is increasing.

Exercise 4.

1) Divide 96 kg into two parts in the ratio of 5:1.

2) Divide 1hr 20mins into two parts in the ratio of 3:2.

3) Divide 256 into three parts in the ratio of 4:3:1

4) A garage charges an extortionate repair bill of £345 for a car service. The ratio of the material costs to labour was 3:5. What was the labour cost?

5) Mortar is made from 70kg of sand to 25kg of cement.
   a) How much sand is needed to mix 35kg of cement?
   b) How much cement is needed to mix with 1500kg of sand?

6) Express the following as ratios in their lowest form:
   a) 30:85
   b) 50:1750
   c) 65:26:39

7) Convert the following to vulgar fractions:

a) 1.25

b) 0.14

c) 0.008

d) 12.56

e) 67.89.

8) 240 students took an examination. 65% of them gained a pass mark, 25% gained a credit whilst the remainder failed. How many students fell into each category?

9) An electrician presents a bill to a customer for £517.79 plus VAT. How much will the customer have to pay if the rate of VAT is:

a) 20%?

b) 5%?

Answers

1) 80:16 kg 2) 48:32 mins 3) 128:96:32 4) £215.63 5) a) 98 kg b) 535.7 kg

6) a) 6:17 b) 1:35 c) 5:2:3

7) a) $\frac{5}{4}$ b) $\frac{7}{50}$ c) $\frac{1}{125}$ d) $\frac{314}{25}$ e) $\frac{6789}{100}$

8) 156 pass; 60 credit; 24 fail

9) a) £621.35 b) £543.68

5: Algebra and transposition of formulae

In this session the student will:

- Gain an understanding of the use of algebra.
- Practice using algebraic rules to transpose formulae.

We now move on to one of the areas that students seem to have most trouble with: that is algebra and transposing formulae.

Without a firm grasp of large areas of background knowledge the electrician will find it almost impossible to achieve the required level of understanding for the subjects they are to study.

Algebra is nothing more than a general description of an arrangement of numbers; a type of coding if you like. Instead of numbers we use letters, and so we start using letters like ***a*** and ***b*** and ***p*** and ***q*** and so on. It can appear to be confusing, but if you remember that we are dealing with real, if unknown, numbers then things are better understood.

You will already have come across algebraic terms in your school life. You will no doubt remember that the area of a rectangle is '***length***' times '***breadth***', or $(l \times b)$. That is an algebraic expression. We are merely using letters to describe a general truth.

The basic laws introduced earlier, **BODMAS** etc. are generalized in algebra.

1. In this instance, it doesn't matter whether there are brackets or not. The addition signs mean that the brackets can all be removed.
$$(a+b)+c=a+(b+c)=a+b+c$$
2. The same thing occurs here. All the signs are effectively multiplication signs and so the brackets can go.
$$(ab)c=a(bc)=abc$$

3. It doesn't matter which letter comes first. This is the same as if you were adding 2 to 3 or 3 to 2; the answer would still be 5.

$$a+b=b+a$$

4. The same thing applies here as it does to the one above. 2 times 3 is the same as 3 times 2, and both equal 6.

$$ab=ba$$

5. Here the letter outside of the bracket is common to either letters or numbers inside the bracket.

$$a(b+c)=ab+ac$$

6. Similarly to the last example, the letter that is the 'denominator' is common to both numerators: it has no particular desire to attach itself to one or the other of the numbers, and must be attached to both.

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

7. Here we have a situation where every letter or number in the first bracket must be multiplied by every other letter or number in the second bracket. The use of the brackets here is the key. Two brackets placed next to each other tell us that they must be multiplied together.

$$(a+b)(c+d)=ac+ad+bc+bd$$

The laws of precedence, which apply to arithmetic, apply also to algebraic expressions. Remember:

BODMAS

1. Evaluate $3ab-2bc+abc$ where $a=1$, $b=3$ and $c=5$.

Replacing a, b, and c with their numerical values gives:

$$(3\times1\times3)-(2\times3\times5)+(1\times3\times5)$$

All you need to do is fill in the numbers where the letters are. Most equations and formulae are set up so that you only have to fill in the numbers.

$$3ab-2bc+abc=9-30+15=-6$$

Try another example.

2. Evaluate $5pq\times 2-pq+2pq$ where, $p=1.5$ and $q=2.25$.

$$5pq\times 2-pq+2pq=5\times 1.5\times 2.25\times 2-1.5\times 2.25+2\times 1.5\times 2.25$$

$$33.75-3.375+6.75=37.125$$

As with the first example, just fill in the numbers and carry out the rules. You should notice that the rules of ***BODMAS*** have been maintained.

Transposition of formulae

Transposition of formulae is the rearranging of equations so that other information can be gathered. It is not necessarily complex but it does need care. Again, once the techniques are learned, it is very easy to come to terms with any, and all types of formulae that you will come across.

The fundamental rules learned with basic operations and with algebra are repeated here. The main rule however is that:

WHAT IS DONE TO ONE SIDE OF THE EQUATION MUST BE DONE TO THE OTHER SIDE.

There are no short cuts and no variations on this!

1. Transpose $R_T = R_1 + R_2$ this formula to make R_1 the subject of the formula.

This is the formula for series connected resistors. The key to this is always to remember that each side of the equation (i.e. each side of the equals sign) must be balanced. To make R_1 the subject of the equation it is necessary to somehow remove R_2. If we subtracted R_2 from both sides, the equation would still be balanced.

Try the principle of adding two simple numbers.

$$3 = 2 + 1$$
$$2 = 3 - 1$$
$$1 = 3 - 2$$

In the above simple example, you can see that $3 = 2 + 1$. The other two variations are also true. The same principle is true in transposing formulae. The only real difference is that we are using letter and number combinations rather than just numbers.

$$R_T = R_1 + R_2$$
$$R_T - R_2 = R_1 + \cancel{R_2} - \cancel{R_2}$$
$$R_1 = R_T - R_2$$

If that seems strange just remember that what is done to one side must be done to another. All things must remain in balance.

In this first example, the number R_2 (and it is a number in that we are using a letter to express an unknown number) is to be removed from the right-hand side of the equation to the left-hand side. As it has been added to another number on the right-hand side, then it must be subtracted from that side to remove it, as the opposite of addition is subtraction.

Try this one.

2. Transpose $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ this so that $\frac{1}{R_2}$ is the subject of the equation.

This is the equation for resistors connected in parallel. We only need to follow the example above.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} - \frac{1}{R_2} = \frac{1}{R_1} + \cancel{\frac{1}{R_2}} - \cancel{\frac{1}{R_2}}$$

$$\frac{1}{R_1} = \frac{1}{R_T} - \frac{1}{R_2}$$

Although it may appear more complex, in reality it is very similar to the previous example.

Notice again that the same value is subtracted from both sides. It matters little if they are whole numbers or fractions the result is the same.

3. The formula that describes power in an electrical circuit is $P = IU$. Transpose this so that I is the subject.

What you need to recognise is that the opposite of multiplication is division and the opposite of division is multiplication.

$$P = IU$$

$$\frac{P}{U} = \frac{I\cancel{U}}{\cancel{U}}$$

$$I = \frac{P}{U}$$

You may be able to see that the letter U has been cancelled out on the right-hand side. You can only cancel out the same letter or number. You can't just remove letters or numbers just because they seem to be in the way.

4. $U = IR$ describes Ohm's Law: transpose it for R

$$U = IR$$

$$\frac{U}{I} = \frac{\cancel{I}R}{\cancel{I}}$$

$$R = \frac{U}{I}$$

Notice again how to get rid of the I on the right-hand side: divide by the I. As long as we do the same to both sides, we are on the right lines. Again, notice the cancelling out of the I in the middle part of the equation.

One more should be enough to make sure that you are able to attempt the examples that follow, and to be familiar with the required techniques when called on.

5. $R = \frac{\rho l}{A}$ describes how resistance is related to length, area and type of material. Transpose it for l.

$$R = \frac{\rho l}{A}$$

$$RA = \frac{\rho l}{\cancel{A}} \times \cancel{A}$$

$$\frac{RA}{\rho} = \frac{\cancel{\rho} l}{\cancel{\rho}}$$

$$l = \frac{RA}{\rho}$$

To begin with, both sides we must multiply both sides by A, to get rid of the A on the right-hand side. The reason for multiplying is that we have initially divided and to cancel out a division we must multiply.

Both sides must then be divided by ρ (***rho***). We now divide because we had initially had a multiplication; again remember to cancel out the ρ on the right-hand side.

Notice that we just had to go through the process twice.

For formulae containing a square term or a square root, things become a little more complex but not impossibly.

Consider the example below.

$s = u^2 t$ transpose to make u the subject.

Move t as you have been previously shown so $u^2 = \frac{s}{t}$

Next square root both sides; $\sqrt{u^2} = \sqrt{\frac{s}{t}}$. A square and a square-root cancel each other out leaving $u = \sqrt{\frac{s}{t}}$.

The reverse also applies; to remove a square-root, square it.

Transpose $Z = \sqrt{R^2 + X^2}$ to make R the subject.

$Z^2 = (\sqrt{R^2 + X^2})^2$	Square both sides
$Z^2 = R^2 + X^2$	Transpose for R
$R^2 = Z^2 - X^2$	Square-root both sides
$R = \sqrt{Z^2 - X^2}$	

Transpose the following:

1. $A = \dfrac{\pi d^2}{4}$ for d

$\left[d = \sqrt{\dfrac{4A}{\pi}} = 2\sqrt{\dfrac{A}{\pi}} \right]$

2. $C = 2\pi r$ for r

$\left[r = \dfrac{C}{2\pi} \right]$

3. $P = I^2 R$ for I

$\left[I = \sqrt{\dfrac{P}{R}} \right]$

4. $P = \dfrac{U^2}{R}$ for U

$U = \sqrt{PR}$

Exercise 5.

Transpose.

1) $Q = It$ for t;

2) $B = \dfrac{\Phi}{A}$ for A;

3) $e = Blv$ for v;

4) $F = BIl$ for B;

5) $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$ for A;

6) $y = mx + c$ for m.

7) If $U = IR$ and $I = 2\,A$, whilst $R = 120\,\Omega$, calculate U.

8) If $e = Blv$, calculate e when $B = 0.2\,T$, $l = 5\,m$ and $v = 25\,ms^{-1}$.

9) If $R_T = R_0(1 + \alpha\theta)$, calculate R_T when $R_0 = 20\,\Omega$; $\alpha = 0.00428 /^{\circ} C$ and $\theta = 30°C$.

6: Indices

In this session the student will:

- Gain an understanding of the use of indices.

Indices relate back to the first section of this study material. It is a means of simplifying numbers quickly and easily. As with so many other areas in maths, there are a number of simple rules to follow.

1. When any number is raised to a power, for example 2^3, the number 2 is called the ***base*** and the ^{3} is the ***index***. Thus $2^3 = 2 \times 2 \times 2 = 8$.

Special names may be used when the indices are 2 or 3, these being "***squared***" and "***cubed***" respectively. You may well have come across these terms before.

2. When no index is shown, then the number (base) is to be raised to the power 1. For example $2^1 = 2$

3. The reciprocal of a number is when you divide 1 by that number. The reciprocal of a number, when provided in index form, is when the index is negative. For example

$$2^{-1} = \frac{1}{2}$$
$$5^{-5} = \frac{1}{5^5}.$$

4. When the base is raised to a fractional index, then the base is 'rooted'. For example

$$2^{\frac{1}{2}} = \sqrt{2}$$
$$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$
$$4^{\frac{2}{3}} = \sqrt[3]{4^2}$$

5. When simplifying calculations involving indices, a number of basic rules or laws can be applied. These are called the ***laws of indices***. These are:

   a) When multiplying two or more numbers together which have the same base, then the indices are be added.

   $$3^2 \times 3^4 = 3^{2+4} = 3^6$$

   b) When dividing two numbers that have the same base, the indices are subtracted.

   $$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

   c). When a number, which is raised to a power, is raised to a further power, the indices are multiplied.

   $$\left(3^5\right)^2 = 3^{5\times 2} = 3^{10}$$

   d). When a number has an index of 0, its value is 1.

   $$3^0 = 1$$

Evaluate

1. $5^2 \times 5^3 = 5^{2+3} = 5^5$
2. $3^2 \times 3^4 \times 3 = 3^{2+4+1} = 3^7$
3. $2 \times 2^2 \times 2^5 = 2^{1+2+5} = 2^8$

Evaluate

1. $\frac{7^5}{7^3} = 7^{5-3} = 7^2$
2. $\frac{5^7}{5^4} = 5^{7-4} = 5^3$

Exercise 7.

1) $3^3 \times 3^4$ 2) $7^2 \times 7^4$ 3) $4^2 \times 4^3 \times 4^4$ 4) $\frac{7^5}{7^2}$ 5) $\frac{12^{25}}{12^{24}}$

6) $\left(7^2\right)^3$ 7) $\left(17^2\right)^4$ 8) $\frac{2^2 \times 2^3}{2^4}$ 9) $\frac{13^5}{13 \times 13^2}$

Express the following in standard form and engineering form to two decimal places:

10) 2,748 11) 33,170 12) 274,218

13) 0.2401 14) 0.0174 15) 0.00923

16) 1,702.3 17) 0.0109 18) 197.62.

7: Trigonometry

In this session the student will:

- Gain an understanding of the use relationship between the sides and angles of a triangle.

Originally trigonometry was the branch of mathematics concerned with solving triangles using trigonometric ratios that were seen as properties of triangles rather than of angles.

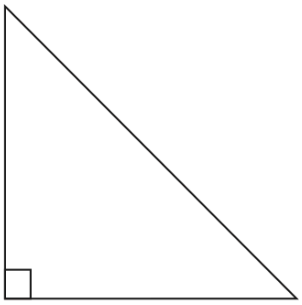
The word Trigonometry comes from a combination of three Greek words:

i) Treis meaning three
ii) Gonia meaning angle
iii) Metron meaning measure.

The Early Greeks developed the subject and used it in a number of very practical ways. Initially it was used in Astronomy but was also used in building. In one famous example a tunnel was dug through a mountain on the Island of Samos, with two teams starting at each end and it met perfectly in the middle.

Later trigonometry was much used in Architecture, Navigation, Surveying and Engineering, but in the last two centuries it has been used more for *Mathematical Analysis* and for repeating *Waves and Periodic Phenomena*.

Consider a right-angled triangle.

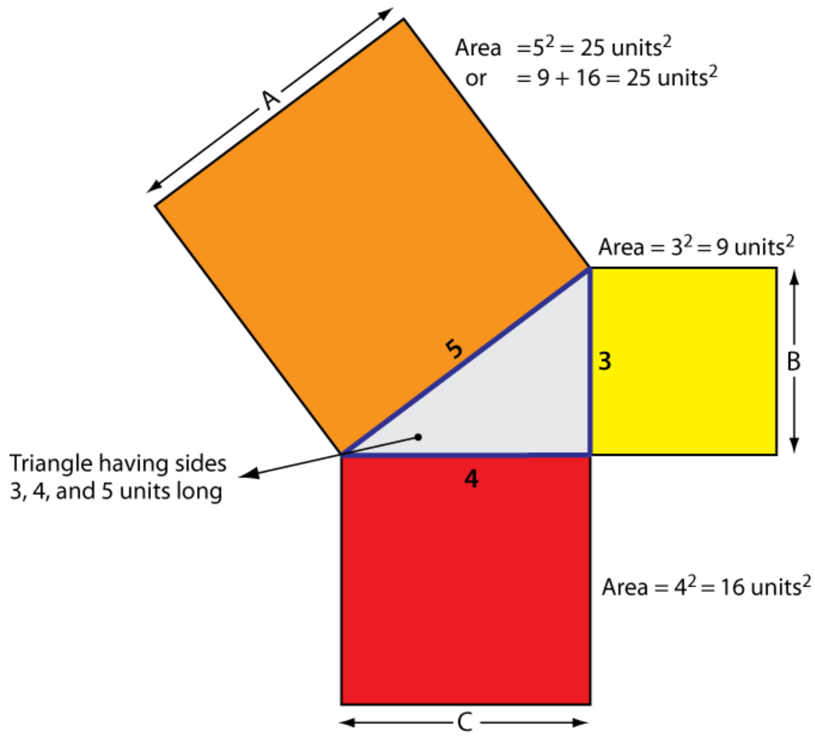


Triangles have a number of qualities. For example, all the angles add up to 180°. The extra qualities of right-angled triangles are of benefit to us in the electrical industry.

A Greek called Pythagoras stated:

"The square on the hypotenuse is equal to the sum of the squares on the other two sides."

So, what does this mean?



Here you can see that a square has been drawn on each of the sides of the triangle. If you were to measure the individual areas, you would find that the two areas of the squares on the shorter sides of the triangle added up to the area of the square on the longest side. The longest side is called the '***hypotenuse***'.

This relationship can and is described using a formula.

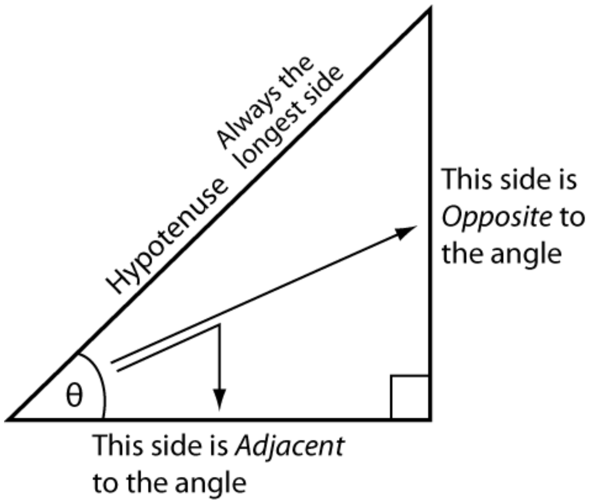
$$a^2 = b^2 + c^2$$

This can be transposed to form a slight variant.

$$a = \sqrt{b^2 + c^2}$$

Simply square-rooted both sides.

The second set of qualities is the relationships between the sides and the angles of a right-angled triangle.



In any right-angled triangle, any angle can be described in terms of a ratio of two other sides. For example, the angle θ (theta) may be found as one of three ratios. These are:

- **tangent** (tan)
- **cosine** (cos) and
- **sine** (sin).

The three ratios are described below.

$$\text{sine } \theta = \frac{\text{opposite side of angle}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{O}}{\text{H}}$$

$$\text{cosine } \theta = \frac{\text{adjacent side of angle}}{\text{hypotenuse}}$$

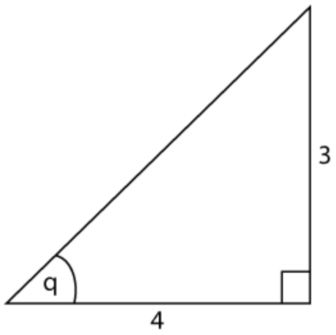
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{A}}{\text{H}}$$

$$\text{tangent } \theta = \frac{\text{opposite side of angle}}{\text{adjacent side of angle}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{O}}{\text{A}}$$

Each of the new terms is a relationship between two sides of the triangle.

1. Based on this triangle determine the values of all the angles and all the sides.



$$a = \sqrt{b^2 + c^2}$$
$$a = \sqrt{3^2 + 4^2}$$
$$a = \sqrt{9 + 16} = \sqrt{25} = \underline{\underline{5}}$$
$$\sin q = \frac{O}{H}$$
$$\sin q = \frac{3}{5} = \underline{\underline{0.6}}$$
$$q = \sin^{-1} 0.6 = \underline{\underline{36.86^\circ}}$$

alternatively,

$$\cos q = \frac{A}{H}$$
$$\cos q = \frac{4}{5} = \underline{\underline{0.8}}$$
$$q = \cos^{-1} 0.8 = \underline{\underline{36.86^\circ}}$$

A few points need to be noticed. Firstly, it doesn't matter which relationship is used; ***sin***, ***cos*** or ***tan***: The angles will still come out the same.

The second point is that the terms ***sin***$^{-1}$ or ***cos***$^{-1}$ or even ***tan***$^{-1}$ all have the same purpose.

What you should do is take your scientific calculator and look at it. You should have a series of buttons that read **sin**, **cos** and **tan**, and usually above them there are three labels showing **sin**$^{-1}$, **cos**$^{-1}$ and **tan**$^{-1}$. These are the buttons used to turn the ratios of the sides into real angles. Just type in your number and press **inv sin** or **2**nd **Fsin**.

Exercise 8.

1) The following right-angled triangles have the following dimensions. Fill in the remainder of the table:

Hypotenuse	Side a	Side b	Angle a	Angle b
7	4			
	5	12		
16		12		
25	16			
	100	150		
230		110		

8: Statistics

In this session the student will:

- Gain an understanding of the use of statistics.

Statistics is generally considered to be a mathematical science concerned with the collection, analysis, interpretation or explanation, and presentation of data. Because statistics has its roots in the gathering of experimental or observed data (empirical) and its focus on applications, statistics is usually considered to be a distinct mathematical science rather than a branch of mathematics.

Because of its complexity this short section will consider just some basic ideas used within statistics.

Tally diagrams

When collecting many readings, it is not very convenient to write down number after number. It is far better to simply 'tally' up the number of readings and then finish of the table later.

Here is a table showing the number of clips in a box. It could just as easily be the values of resistors or types of capacitors.

Variable (x)	**Tally marks**	**Frequency (f)**
96	//	2
97	///	3
98	//	2
99	/////	5
100	//////////	10
101	///////	7
102	/////	5
103	//	2
104	/	1
	Sum of the frequency $\sum f$	**37**

The strange symbol, $\sum$, just means the sum of, or adds them all together.

You can see that it would be much easier to tally up and then complete the table. The data is considered to be discrete. You can see this because there is no real continuity. Each box of clips has a set amount that can be measured.

What about measuring the resistances of carbon resistors all rated at 1kΩ. The following readings were found and recorded in a table.

Variable (x)	Tally marks	Frequency (f)
801-850	///	3
851-900	////////	8
901-950	////////////	12
951-1000	////////////////	16
1001-1050	///////////////////////	23
1051-1100	////////////////	16
1101-1150	//////////	10
1151-1200	///////	7
1201-1250	/////	5
	Sum of the frequency $\sum f$	**100**

This data is continuous data. You can see that it forms part of a continuous scale and that the measurements of resistance depend on the accuracy of the test instrument.

The data in the table above has been grouped; this helps to keep the data in a form that is easily manageable.

Now that we know how to collect and separate that data into continuous or discrete values, we now need to consider what to do with all this data. It is probably worthwhile just recognising the difference between information, data and datum.

Datum is singular and relates to a single item.
Data is plural and relates to the values gathered.
Information is the useful product of the collected data.

Now we need to deal with the data.

Mode

Quite simply the mode is the most commonly occurring value. This is straightforward with discrete data.

Variable (x)	**Tally marks**	**Frequency (f)**
96	//	2
97	///	3
98	//	2
99	/////	5
100	//////////	10
101	///////	7
102	/////	5
103	//	2
104	/	1
	Sum of the frequency $\sum f$	**37**

Here is the table from earlier. It is clear that the most commonly occurring value is 100; occurring ten times. Remember that it is the most commonly occurring value that is the mode, not the most common frequency.

There can be instances where the mode is more than one value. If the value 101 had a frequency of 10, then that would also be the mode, and we would have a mode of 100 and 101.

As an example, from the following lists of numbers, state what the mode is/are.

1. 52 51 52 59 60 58 54 53 50.

2. 20 28 44 32 30 28 30 26 28 34.

3. 51 43 30 36 42 40 27 31 61 57 74
   64 55 42 83 52 50 37 41 71.

Remember that the mode is the most commonly occurring value.

Answers

1) 52
2) 28
3) 42

Mean

The ***mean*** of a series of data is the ***average*** value.

Look at the following values of voltage.

120 V

134 V

134 V

140 V

145 V

150 V

165 V

120 V

210 V

230 V.

Here there are ten voltages. The mode would be 134 V and 120 V as they are the most commonly occurring values; both occurring twice. The mean however, is the average of all ten values.

$$\text{Mean} = \frac{\text{Values}}{\text{Number of readings}}$$

$$\text{Mean} = \frac{120+134+134+140+145+150+165+120+210+230}{10}$$

$$\text{Mean} = \frac{1548}{10} = 154.8V$$

As an example, determine the mean of the following sets of numbers.

1)	52	51	52	59	60	58	54	53	50.			
2)	20	28	44	32	30	28	30	26	28	34.		
3)	51	43	30	36	42	40	27	31	61	57	74	64
	55	42	83	52	50	37	41	71.				

Answers

1) 54.3
2) 30
3) 49.35

Median

We have seen that the mode is the most commonly occurring value; that the mean is the average of the values, and now we have a third aspect introduced. The ***median*** is the ***middle*** value.

Let's consider our ten voltages.

120 V

134 V

134 V

140 V

145 V

150 V

165 V

120 V

210 V

230 V.

To determine the median value we need to place them in order.

120 V
120 V
134 V
134 V
140 V
145 V
150 V
165 V
210 V
230 V

In ten readings, the middle value is split between reading five and six, or 140 V and 145 V. This means that the middle reading has to be averaged:

$$\text{Median} = \frac{140+145}{2} = \frac{285}{2} = 142.5V$$

If there had been eleven readings (any odd number will do) then it would have been more straightforward to choose the mid-reading.

As an example, from the following lists of numbers, state what the median is.

1)	52	51	52	59	60	58	54	53	50.			
2)	20	28	44	32	30	28	30	26	28	34.		
3)	51	43	30	36	42	40	27	31	61	57	74	64
	55	42	83	52	50	37	41	71.				

Answers

1) 53
2) 29
3) 46.5

At this level of study, there is no need to into any more detail about any of the three terms we have introduced so far.

Charts and diagrams

The use of charts and diagrams is very common in statistics. They show values and shapes that are not immediately recognised from a table or list.

There are some questions you have to ask yourself before you embark on simply drawing up anything.

1) What am I trying to show?
2) What is the best way to express the data I am trying to show?

The drawing of charts is where maths meets communication. They are used to clarify certain points, and you cannot expect one chart to do everything.

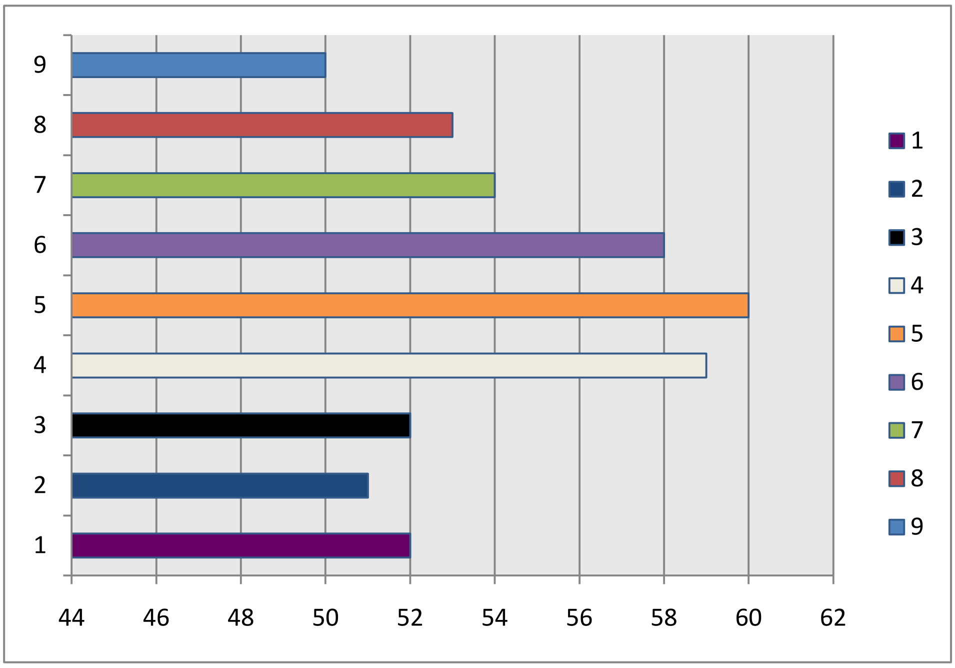
Over the next few pages I will use the three sets of numbers from the examples you have just looked at for mode, mean and median, and present that information in terms of some basic bar and pie charts.

My charts have been drawn using a spreadsheet. There is nothing stopping you drawing them by hand if that is what you are most comfortable with.

Bar charts

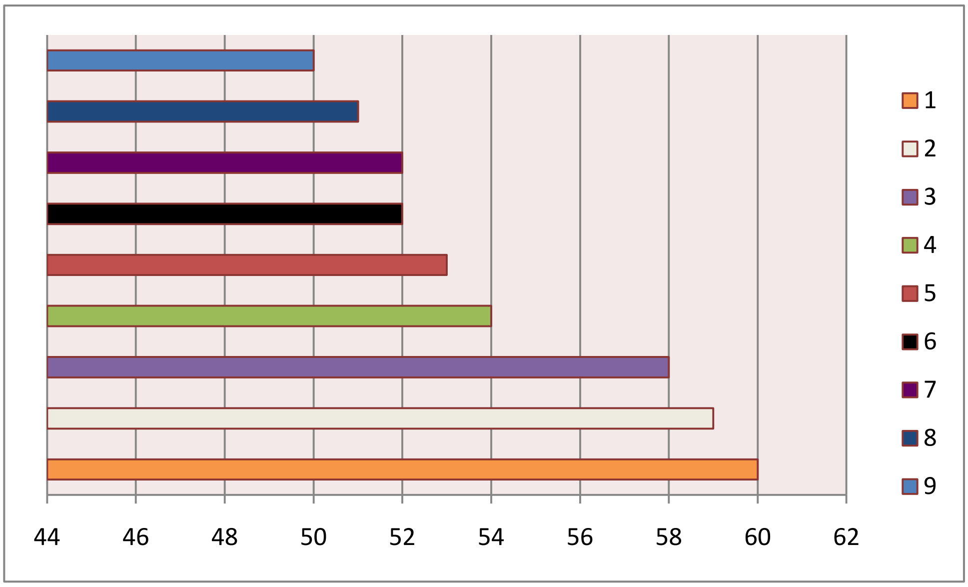
We'll look at each set of numbers in turn and address some of the issues encountered.

1) 52 51 52 59 60 58 54 53 50.



In the chart above, I have simply labelled the 'x' axis, as there is no information regarding what the numbers represent. I have then drawn bars across to represent whatever the numbers mean. This should not be new to you. Notice that the scale does not need to start at zero (0). This enables the differences to be more easily seen. You should also note that there is no particular order to what I have done.

The chart over the page shows the same data in ascending order.



You can see that the graph look very different, but in reality the same data is being shown.

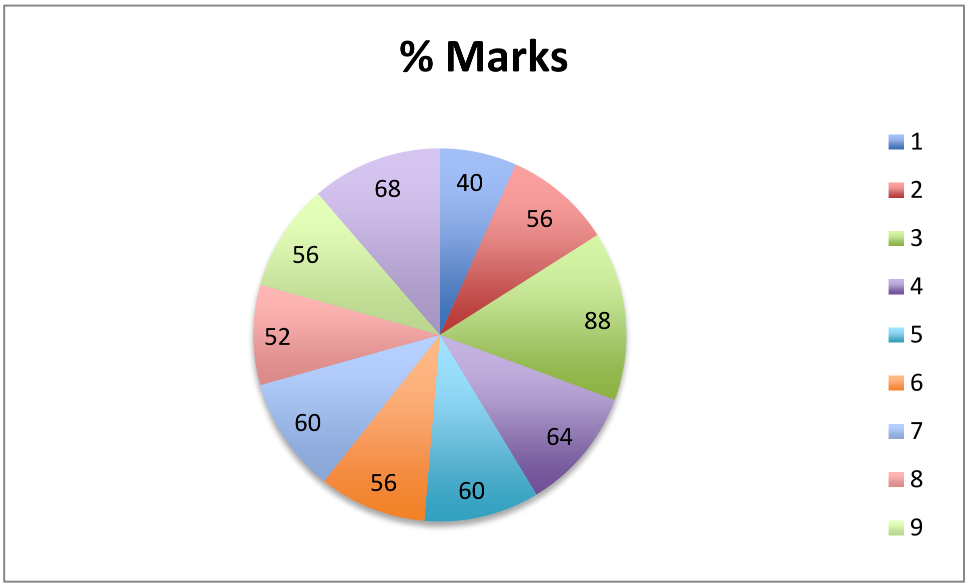
With a straightforward bar chart, you can quickly see the mode and it is can also be easier to determine what the median value will be.

There are other ways however, in which data can be represented.

Pie chart

Here, I will make use of the set of numbers in the second question. I will assume they represent the marks gained by student in a test where the maximum mark was 50.

2) 20 28 44 32 30 28 30 26 28 34.



The above chart shows how data may be expressed in a way that does not necessarily immediately show huge differences. Hopefully you can see that without the numbers showing the marks attached to the chart you would have had difficulty determining which was the largest sized slice of pie.

In reality, pie charts are only useful for highlighting quite large differences between data.

There are many other types of chart, and the better the spreadsheet, the greater the options.

Exercise 9.

1) The percentage marks gained by a group of students in a phase test were as follows:

23	87	90	55	67	50	34	56	76	87	23
45	40	45	46	96	33	44	56	78	12	34
66	46	52	51	56	69	72	66	77	91	22
65	45	46	47	75	65	34	42	76	89	77

a) What is the range? [**difference between maximum and minimum mark**]

b) What is the mean?

c) What is the mode?

d) What is the median?

e) Create a frequency table based on 10% variations;

f) Draw an appropriate type of chart to express the number of students gaining marks within those bands.

Answers.

Exercise 2

1)	42	2)	19	3)	30	4)	14	5)	11
6)	16	7)	5	8)	5	9)	15	10)	29
11)	-30	12)	124	13)	0.0972	14)	504	15)	4
16)	44	17)	27	18)	81	19)	4	20)	1.8

Exercise 3

1)	$\frac{9}{10}$	2)	$\frac{23}{63}$	3)	$\frac{17}{21}$	4)	$\frac{5}{63}$	5)	$\frac{47}{63}$
6)	$1\frac{16}{21}$	7)	$8\frac{51}{52}$	8)	$\frac{5}{12}$	9)	$11\frac{80}{209}$	10)	$1\frac{7}{20}$
11)	$\frac{5}{12}$	12)	$\frac{8}{15}$	13)	$1\frac{1}{6}$	14)	$2\frac{28}{55}$	15)	$\frac{28}{87}$
16)	$\frac{9}{143}$								

Exercise 4

1)	$\frac{3}{10}$	2)	$\frac{1}{1000}$	3)	$\frac{1}{2}$	4)	$1\frac{1}{20}$	5)	$2\frac{3}{50}$
6)	$\frac{1}{8}$	7)	$\frac{2}{25}$	8)	$1\frac{9}{20}$	9)	$\frac{4343}{10000}$	10)	$7\frac{7}{100}$

Exercise 5

1) 80:16 kg 2) 48:32 mins 3) 128:96:32 4) £215.63

5) a) 98 kg b) 535.7 kg

6) a) 6:17 b) 1:35 c) 5:2:3

7) a) $\frac{5}{4}$ b) $\frac{7}{50}$ c) $\frac{1}{125}$ d) $\frac{314}{25}$ e) $\frac{6789}{100}$

8) 156 pass; 60 credit; 24 fail 9) a) £621.35 b) £543.68

Exercise 6.

1) $t = \frac{Q}{I}$ 2) $A = \frac{\Phi}{B}$ 3) $v = \frac{e}{Bl}$ 4) $B = \frac{F}{Il}$ 5) $A = \frac{Cd}{\varepsilon_0 \varepsilon_r}$

6) $m = \frac{y - c}{x}$ 7) 240 V   8) 25 V   9) 22.57 Ω

Exercise 7.

1) 3^7 2) 7^6 3) $4^9 = 2^{18}$ 4) 7^3 5) 12

6) 7^6 7) 17^8 8) 2   9) 13^2

10) $2.75 \text{x} 10^3$ = 2.75k   11) $3.32 \text{x} 10^4$ = 33.17 k

12) $2.74 \text{x} 10^5$ = 274.22 k   13) $2.40 \text{x} 10^{-1}$ = 24.1 m

14) $1.74 \text{x} 10^{-2}$ = 17.4 m   15) $9.23 \text{x} 10^{-3}$ = 9.23 m

16) $1.70 \text{x} 10^3$ = 1.7 k   17) $1.1 \text{x} 10^{-2}$ = 10.9 m

18) $1.98 \text{x} 10^2$ = 0.20 k

Exercise 8.

Hypotenuse	**Side a**	**Side b**	**Angle a**	**Angle b**
7	4	**5.74**	**38.85**	**55.15**
13	5	12	**22.62**	**67.38**
16	**10.58**	12	**41.41**	**48.6**
25	16	**19.21**	**39.8**	**50.2**
180.28	100	150	**33..69**	**56.31**
230	**202**	110	**61.43**	**28.57**

Exercise 9

a) Maximum reading is 96%

Minimum reading is 12%

Range = Max-Min = 96-12 = 84%

b) Mean = 56.96%

c) Mode = 44%

d) Median = 55.5%

Remember that for an even number of results we have to average the two middle values.

e)

Marks %	**Frequency**
0-10	0
11-20	1
21-30	3
31-40	5
41-50	10
51-60	6
61-70	6
71-80	7
81-90	4
91-100	2
No. of readings	**44**

f)