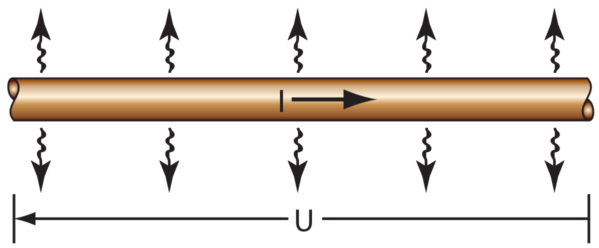
Level 3 Diploma
in
Installing Electrotechnical Systems & Equipment

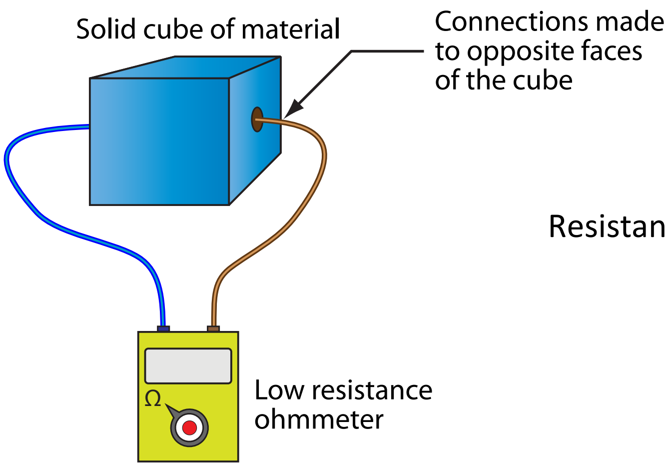
C&G 2357

Unit 309-4 Understand the relationship between resistance, resistivity, voltage, current and power

$$\textit{Power consumed} = \textit{Voltage applied} \times \textit{current flow}$$

$$P = U \times I$$





$$\text{Resistance} \propto \frac{\text{Length}}{\text{cross sectional area}} = \frac{l}{a}$$

$\propto = \rho$; the constant of proportionality

$$R = \frac{\rho \times l}{a}$$

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Learning Materials for the Electrical Industry

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Aims and objectives

By the end of this study book you will have had:

- Describe the basic principles of electron theory.
- Identify and differentiate between good and poor conductors.
- State the types and properties of cables.
- Describe what is meant by resistance and resistivity.
- Explain the relationship between current, voltage and resistance in series and parallel circuits.
- Calculate the values of current, voltage and resistance in series and parallel circuits.
- Calculate the values of power in series and parallel circuits.
- Describe voltage drop.
- Describe the chemical and thermal effects of electric current.

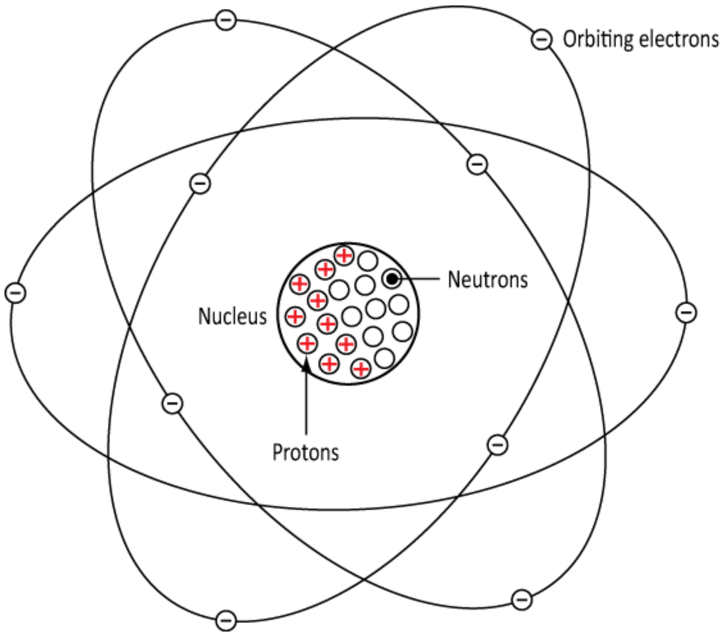
1: Electrons and charge

In this session the student will have the opportunity:

- Describe the basic principles of electron theory.

To understand about electricity we have to understand how it moves along a cable. To do this we have to look at electron theory.

Everything around us is made up of atoms. As far as this level of electrical study is concerned, an atom is made up of a number of different particles. These particles are called '***electrons***', '***protons***' and '***neutrons***'.



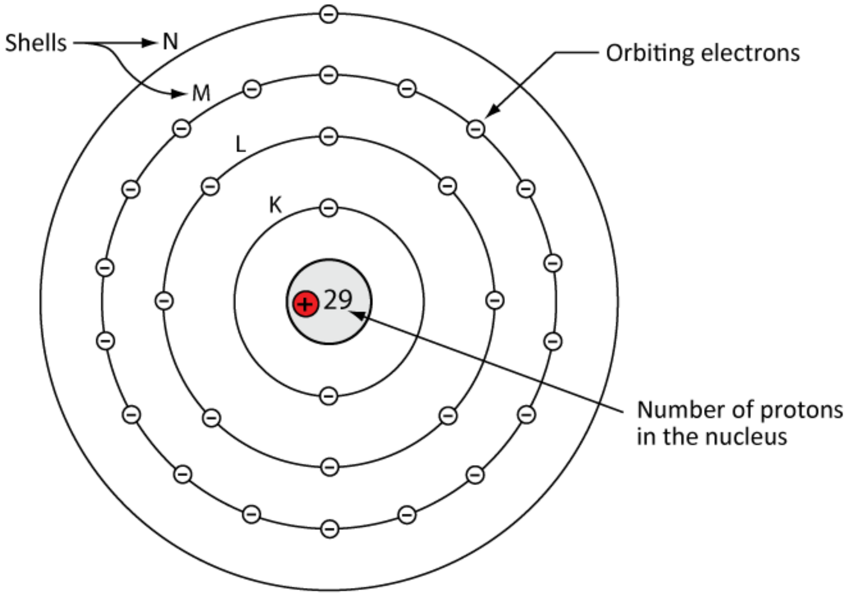
From the diagram above, you can see that the atom looks a little like a miniature solar system.

The centre of the atom is called the '***nucleus***'. The nucleus contains a number of protons with the same number of neutrons. For our purposes, we can ignore the neutrons.

The protons have a **positive** electrical charge. Around the nucleus of the atom, there are a number of orbiting '***electrons***'. These electrons have a **negative** electrical charge.

The electrons are held in their orbit by the attraction that each of the electrical charges has for one another (Protons attract Electrons). The distance away from the nucleus that each electron orbits varies according to the amount of energy that an electron has.

The electrons are said to orbit in levels that are given labels of K, L, M, and N.



An atom of copper

The number of electrons that inhabit each of these levels, or '***shells***', is set. However, the electrons are constantly moving from one level to another depending on their energy level. The diagram above shows the shell structure of a copper atom.

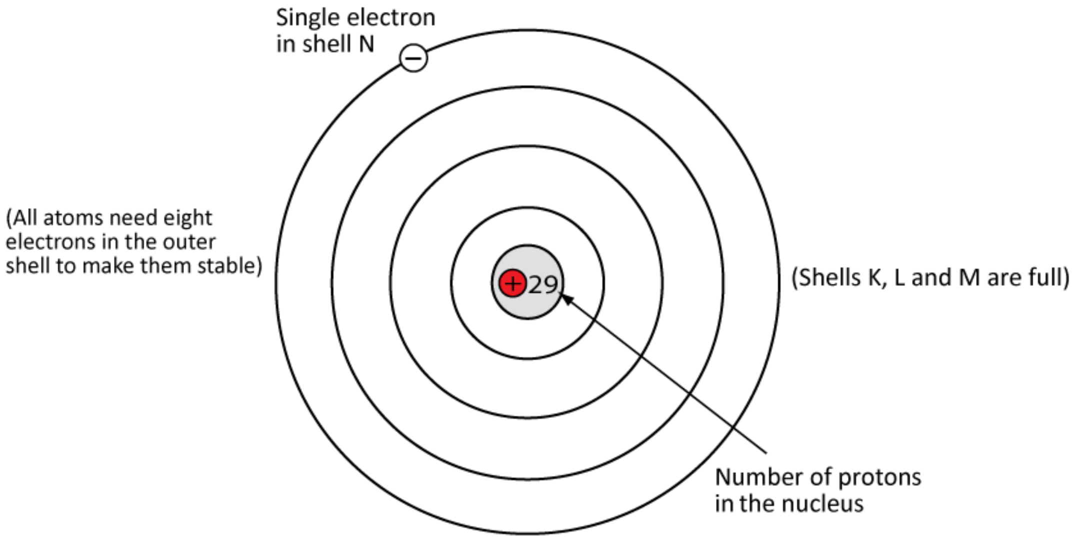
Below is a table showing the numbers of electrons that inhabit each energy level.

Shell	Shell number	Maximum number of electrons in orbit
K	1	2
L	2	8
M	3	18
N	4	32

In general the number of electrons in a shell is given by $2n^2$ where n is the shell number.

The important thing to remember however is that electrons and protons have an electrical charge, and that if there is an imbalance between the two, then the atom has become electrically charged. This atom is then called a positive or negative '***ion***'.

When an atom has energy applied to it, it will either gain an electron or lose an electron. This happens when only one or two electrons are orbiting in the outer shell.



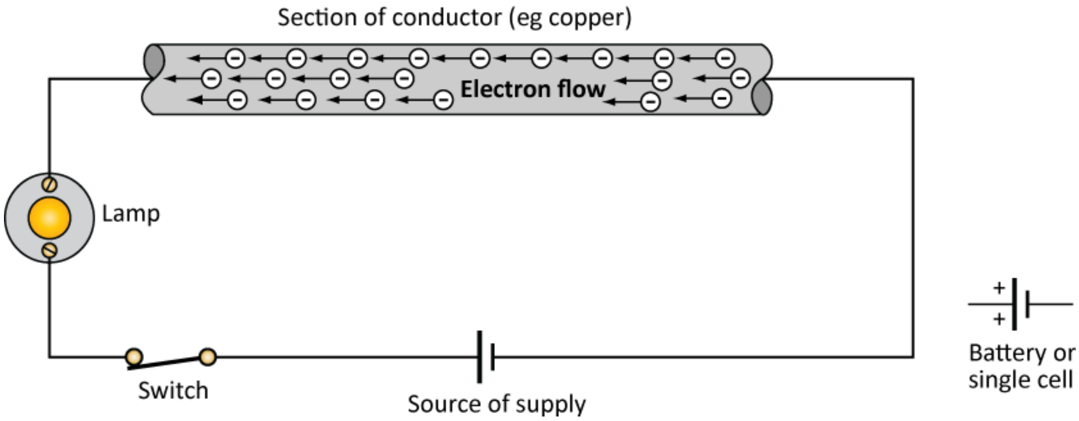
An atom of copper

As can be seen from above; for the copper atom the outer shell only contains the odd electron. It is very easy for only small amounts of energy to displace these electrons. This is why copper is such a good conductor.

Current flow

We have looked at atoms, electrons, and protons. What does this have to do with electricity?

Electricity can be defined as a flow of electrons. When a voltage is applied to a conductor then electrons are provided with energy, this energy forces electrons to move out of one orbit into the orbit of a neighbouring atom. This continues all the way down the conductor. This flow of electrons is called current or current flow.

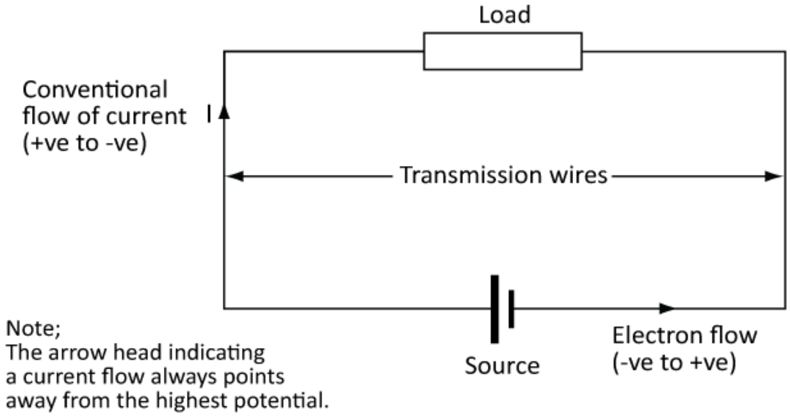


For current to flow there are two important criteria to be met. These are:

- there must be a complete circuit.
- there must be a force applied to '*drive*' the electrons around the circuit.

You can see from above, that for current to flow there must be a source of energy (generator, battery etc.), a load (lamp etc.) and some means of transmission (wires etc.) the energy.

Notice the direction of the current flow. What is termed '***conventional***' current flow, starts at the positive terminal and flows around the circuit and returns to the negative terminal. Now, because we know that current flow is a flow of electrons, we see that current actually flows from the negative to the positive terminal. This is called the '***electron***' flow.



You have already seen the s.i. unit for current is the ampere (amp). The definition of current is fairly simple.

> Current is the rate at which electricity flows around a circuit.

The '***ampere***' is often shortened to the '***amp***'.

So, how do we measure electricity? To count the number of electrons is not sensible as it would be very large. The unit of '***electrical quantity***' is the '***coulomb***' (koo-lom). The definition of the coulomb is:

> The quantity of electricity passing a certain point in a circuit when a current of 1 amp is maintained for 1 second.

You can try to understand the term coulomb by thinking of it as a package of electricity passing a particular point in a circuit.

We are now coming to one of those times when a formula is in order.

If the coulomb is the quantity of current passing a point in a circuit for a certain time, the two things necessary are current and time.

$$Q = It \quad \text{or} \quad Q = I \times t$$

$$\begin{aligned} \text{where} \quad & Q = \text{quantity of electricity } (\text{C}) \\ & I = \text{current } (\text{A}) \\ & t = \text{time } (\text{s}) \end{aligned}$$

Try the example below.

1). A current of 5 A flows for a period of 2 minutes. How much charge is passing through the circuit?

Remember before starting any question, it is important that you have turned everything into base units.

$$\begin{aligned} Q &= It \\ Q &= 5 \times 120 \\ Q &= \underline{\underline{600C}} \end{aligned}$$

Notice that I have converted time into seconds. There are 60 seconds in a minute and there are 2 minutes. This gives us 120 seconds of time.

2). A charge of 12 C passes a point in a circuit in 0.3 s. What is the current in the circuit?

With this one, remember that you are going to have to transpose the formula.

Once the formula has been transposed then all you have to do is fill in the numbers.

$$Q = It, \; so \;\; I = \frac{Q}{t} \quad then \;\; I = \frac{12}{0.3} \; \therefore I = \underline{\underline{40A}}$$

Exercise 1.

1) Give a definition of current, in your own words.

2) What is a free electron?

3) What is the quantity of electricity measured in?

4) If 3 A flow in a circuit for 2 hours what will be the charge?

5) For a charge of 900 C and a current of 2 A what will be the time in minutes?

6) The time taken for a battery to charge is 4 hours, what will be the charge if a current of 4 A flows?

7) How many electrons can inhabit the innermost shell of an atom and what label is attached to that shell?

8) What is an ion?

9) In which direction does conventional current flow?

10) In which direction do electrons flow?

2: Conductors and Insulators

In this session the student will:

- Gain an understanding of the difference between conductors and insulators.

This session is not going to look at cables as in the types available (that is addressed in a different part of the course), but more into what makes for a good conductor and a good insulator.

We have just looked at atoms and you have learnt that all atoms have a nucleus and orbiting electrons. The electrons of different types of atoms have different degrees of freedom to move around. With some types of materials, such as metals, the outermost electrons in the atoms are so loosely bound that they randomly move in the space between the atoms of that material by nothing more than the influence of room-temperature heat energy. Because these virtually unbound electrons are free to leave their respective atoms and float around in the space between adjacent atoms, they are often called *free electrons*.

In other types of materials such as glass, the atoms' electrons have very little freedom to move around. While external forces such as physical rubbing can force some of these electrons to leave their respective atoms and transfer to the atoms of another material, they do not move between atoms within that material very easily.

This relative mobility of electrons within a material is known as electrical *conductivity*. Conductivity is determined by the types of atoms in a material and how the atoms are linked together with one another. Materials with high electron mobility (many free electrons) are called *conductors*, while materials with low electron mobility (few or no free electrons) are called *insulators*.

Listed here are a few common examples of conductors and insulators. It is likely that at some point in your career you will come into contact with the majority of them:

Typical conductors include:

- silver
- copper
- gold
- aluminium
- iron
- steel
- brass
- bronze
- mercury
- cast iron
- graphite
- dirty water
- concrete

Different metals are used for a variety of reasons that relate to their specific properties. These will include how good they are at allowing current to flow, but also will include the strength, malleability, density and their ability to withstand chemical attack.

Insulators

Although there are many types of material that are good insulators, the insulators that we use in electrical work has to have a range of properties which will include flexibility, lifespan as well as good electrical insulating properties.

There are a variety of types of insulators used in electrical work. These are commonly:

- glass
- rubber
- oil
- fiberglass
- porcelain
- ceramic
- (dry) paper
- plastic
- air

It must be understood that not all conductive materials have the same level of conductivity, and not all insulators are equally resistant to electron motion.

For instance, silver is the best conductor in the "conductors" list, offering easier passage for electrons than any other material cited. Dirty water and concrete are also listed as conductors, but these materials are substantially less conductive than any metal.

Physical dimension also impacts conductivity. For instance, if we take two strips of the same conductive material -- one thin and the other thick -- the thick strip will prove to be a better conductor than the thin for the same length. If we take another pair of strips -- this time, both with the same thickness but one shorter than the other -- the shorter one will offer easier passage to electrons than the long one. This is like water flowing in a pipe; a fat pipe offers easier passage than a skinny pipe, and a short pipe is easier for water to move through than a long pipe, all other dimensions being equal.

There is a formula for this and will be dealt with more fully in the next session.

It should also be understood that some materials experience changes in their electrical properties under different conditions. Glass, for instance, is a very good insulator at room temperature, but becomes a conductor when heated to a very high temperature.

Gases such as air, normally insulating materials, also become conductive if heated to very high temperatures. Most metals become poorer conductors when heated, and better conductors when cooled. Many conductive materials become perfectly conductive (this is called *superconductivity*) at extremely low temperatures.

If we want electrons to flow in a certain direction to a certain place, we must provide the proper path for them to move, just as a plumber must install piping to get water to flow where he or she wants it to flow. To facilitate this, *wires* are made of highly conductive metals such as copper or aluminium in a wide variety of sizes. To protect the consumer from electric shock, they are insulated with a material from the list above depending upon the environmental conditions present.

Exercise 2.

1) Using the college internet or the library, state a typical use for each of the conductors and insulators listed above. Include in your list any particular properties the conductors or insulators may exhibit.

   For instance, copper is malleable, cast iron is brittle etc.

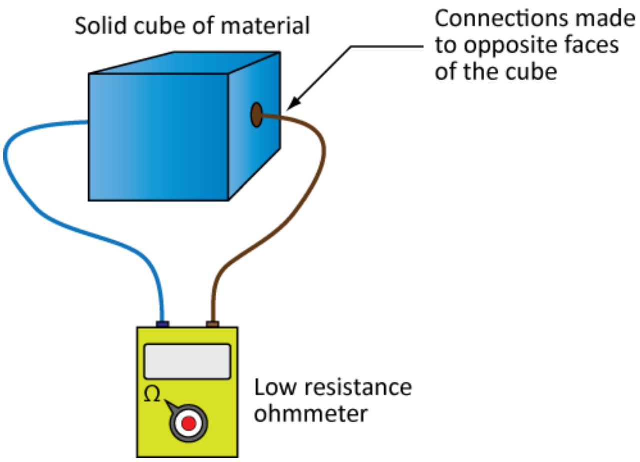
3: Resistivity

By the end of this session you will have had the opportunity to understand what resistance is and what it depends on.

In the previous session it was stated that some conductors are more conductive than others. Each conductor or insulator for that matter has a resistivity value. With this figure and a formula, it is possible to determine how good a conductor it will be by looking at what its overall resistance is, when we consider its length and cross-sectional area.

Therefore, we are going to consider something definitely electrical – resistance. We want to know what it is and how it can be determined. We will spend some time later in this unit looking at resistance and its relationship to voltage and current as defined in Ohm's law. However for the present we need to be aware that resistance is measured in *Ohm's* with a unit label of *Ω*.

When any material has a supply connected to it, it has a particular resistance. This particular resistance is called the '***resistivity***' of the material or its '***specific resistance***'.



Below is a table of many of the relevant values of resistivity for common materials.

Material	ρ (Ωm)	Material	ρ (Ωm)
Copper	1.67×10^{-8}	Glass	1×10^{5}
Silver	1.6×10^{-8}	Magnesium oxide	1×10^{7}
Lead	20.65×10^{-8}	Perspex	1×10^{13}
Aluminium	2.65×10^{-8}	Silicon	5×10^{2}
Tin	12.8×10^{-8}	Carbon	1×10^{-5}
Solder	40.5×10^{-8}	Steel	12×10^{-8}

These values are set for a specific temperature. Once the temperature varies then the resistance values vary, and so any figure needs to be quoted at a certain temperature.

Looking at resistance, what we have is a situation where if you have a particular material it will have a particular ***specific resistance***. If the length of that material is increased, it will also change its resistance, in fact increase the length and you increase the resistance by that same proportion. If that same material changes in size, or area, then it will change its resistance.

Increasing the area reduces the resistance.

What we now have is the ability to formulate an equation that can be used.

ρ (***rho***) is the symbol used for resistivity

$$R=\frac{\rho l}{A}$$

Where $R=\text{resistance}\ (\Omega)$

$\rho=\text{resistivity}\ (\Omega m)$

$l=\text{length}\ (m)$

$A=\text{area}\ (m^2)$

Increasing the length increases the resistance.

Increasing the area decreases the resistance

Specifically, resistance is a measure of the degree with which a conductor opposes the passage of electric current.

The general rule that you need to remember when working out any problem with regard to resistivity is to make sure that the units that you use are consistent. What is meant by this is that if an area is given in mm^2, then the length must also be in mm^2 and the resistivity must be in Ωmm^2. You could always use m^2 at which point everything else would need to be converted to m^2.

Follow the example below, and you will see what is meant.

1/. A length of copper conductor 55 m long, having an area of 2.5 mm^2 has a specific resistance of 1.67×10^{-8} Ωm. What is its resistance?

$$R=\frac{\rho l}{A}$$

$$R=\frac{1.67\times10^{-8}\times55}{2.5\times10^{-6}}$$

$$R=\frac{0.9185\times\cancel{10^{-6}}}{2.5\times\cancel{10^{-6}}}$$

$$R=\underline{\underline{0.3674\Omega}}$$

Remember it is always best to decide beforehand whether you are going to convert everything to metres or to millimetres. In this instance, I will change everything to metres and m^2.

Try this next one.

2). Compare the resistance of two conductors; one copper the other aluminium. The area of each is 120 mm^2 and the length of run is 125 m. Assume that the resistivity of copper is 1.67×10^{-8} Ωm and aluminium is 2.65×10^{-8} Ωm.

Again, make sure that everything is converted into the same units. I will convert the area again.

$$R_{Copper}=\frac{\rho l}{A}$$

$$R=\frac{1.67\times10^{-8}\times125}{120\times10^{-6}}$$

$$R=\frac{2.0875\times\cancel{10^{-6}}}{120\times\cancel{10^{-6}}}$$

$$\therefore R_{Copper}=\underline{\underline{0.0174\Omega}}$$

$$R_{Aluminium}=\frac{\rho l}{A}$$

$$R=\frac{2.65\times10^{-8}\times125}{120\times10^{-6}}$$

$$R=\frac{3.3125\times\cancel{10^{-6}}}{120\times\cancel{10^{-6}}}$$

$$\therefore R_{Aluminium}=\underline{\underline{0.0276\Omega}}$$

It can be seen that the better conductor is the copper one, no surprise there.

Exercise 3

1) A coil has an effective length of 12 m. What will be the resistance of the coil if its area is 0.25 mm^2 and its resistivity is 1.67×10^{-8} Ωm?

2) If the length of a conductor halves and the area doubles, what will happen to the resistance?

3) A conductor of length 40 m is extended to 80 m with no increase in size. If the initial resistance is 1.2 Ω what will be the new resistance of the extended cable?

4) The light in a car park is supplied via a cable 85m long. The area of the cable is 6mm^2. Calculate the resistance of the cable if the resistivity is 1.67×10^{-8} Ωm.

5) A coil is made up of 10 000 turns of copper wire. The diameter of the former around which the wire is wound is 39.8 mm, and the area of the conductor is 0.275 mm^2. Calculate the resistance of the coil if the resistivity is 1.67×10^{-8} Ωm. [Hint: remember how to calculate the circumference of a circle]

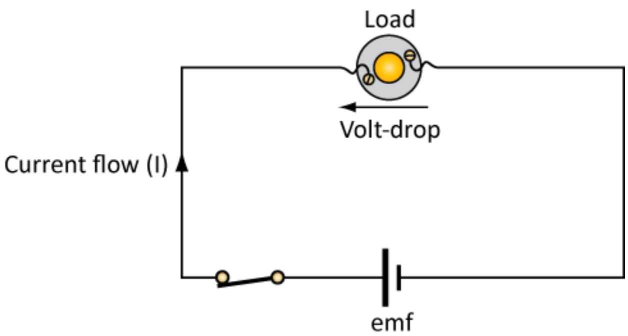
4: Ohm's law, resistance and voltage

In this session the student will:

- Gain an understanding of resistance and its relationship to voltage.

We have already looked at the definition of current, and seen that it can be simplified to '***a flow of electrons***'. The question that leads on from this is, '***What makes the current flow?***'

Earlier, we mentioned that any electrical circuit requires three things, namely a source, a load and a means of transmission.

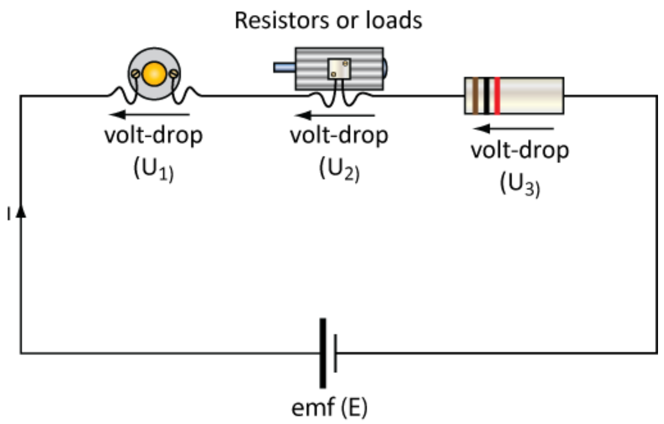


In this simple circuit we have a source (the cell), a load (that which we want to use) and transmission wires linking the source to the load. As we only have one load the value of the emf will be the same as the voltage that we would measure across the load.

When we measure the voltage dropped across the load we don't call this an emf but rather a ***volt drop*** or ***potential difference***.

It may seem to be precise to highlight the differences but it is important. The emf provides the force that drives the current around the circuit, this circuit may have any number of loads connected to it, at which point the emf would not be the same as the voltage dropped but would be made up of all the voltages dropped across the loads.

Below we can see that the voltages dropped across each load or resistors are different to the emf. When they are combined however, they do add up to the same value as the source emf.



$$E = U_1 + U_2 + U_3$$

Although emf and volt drop or potential difference, are measured in volts, the symbol is different, and it is important that you start to use the correct terminology. You must start to talk the electrical language.

You will use ***U*** more than you will use ***E***. ***E*** is used when we are looking at the voltage source such as a battery or a generator, whilst ***U*** is used to describe the volt drop or the potential difference. You may come across books and people who still use ***V***. This is not a great surprise as ***V*** has been used for a long time. You are still permitted to use either U or V as a label, but U is preferred.

We come now to the most important formula that you are going to come across. It is simple and yet is used in so many seemingly different situations. It is called ***Ohm's Law***.

In 1827 Georg Simon Ohm discovered that if you applied an emf to a metal, maintained at a constant temperature, of a set length and cross-sectional area (csa), then, as you varied the voltage the current would vary in the same proportion.

Simply put, he stated that voltage was proportional to current or $U \propto I$, where $(\propto)$ means proportional to, when the temperature of a metal was held constant. He also wondered what it was that linked the voltage and the current, and this link was the ***resistance*** of the conductor.

If you can picture in your mind's eye a hosepipe; resistance to the flow of water would occur when you squeezed the pipe. The more the pipe was squeezed the greater would be the opposition to the flow of water. It is this opposition to the flow of water that is similar to the idea of electrical resistance.

Ohm's Law can be remembered in a number of more useful formats and the most common are:

$$U = IR \quad I = \frac{U}{R} \quad R = \frac{U}{I}$$

It is essential that you remember Ohm's Law, and are confident enough to use it in a wide variety of situations. Follow these examples.

1). The insulation resistance between a line conductor and earth is 2 MΩ. If the supply voltage is 230 V what will be the leakage current?

$$U = IR \quad \text{transpose}$$
$$I = \frac{U}{R}$$
$$I = \frac{230}{2 \times 10^6}$$
$$I = \underline{\underline{115 \mu A}}$$

2). A current of 60 mA flows in a circuit through a resistance of 20 kΩ. What is the voltage dropped across the resistor?

Again, do not get confused by the terms used. You should already be familiar with volt drop, and recognise that all that is being asked for is the voltage.

$$U = IR$$
$$U = 60 \times 10^{-3} \times 20 \times 10^3$$
$$U = \underline{\underline{1200V}}$$

Exercise 4.

1) Using just Ohm's Law fill out the table below.

U	230		110	400		230	230	50
I	12	14		63	3	5		10
R		15	77		65		9	

2) A 1 kΩ resistor is placed into a circuit where the supply voltage is 6 V, what current will flow?

3) A circuit contains two loads connected in series. The volt-drop across one of the loads was measured as being 1.8 V @ 30Ω. The supply voltage was measured as being 6 V, if the current flowing is 60 mA, what is the resistance of the unknown load?

This question is for the more able student!

4) The current of Q3 is to be reduced by 40% by means of connecting a resistor into the circuit. Using Ohm's law, determine the value of this resistor.

*This is quite a difficult question; I would recommend drawing a simple sketch of the new arrangement. This will have three components in series; the original load that had 1.8 V dropped across it, an existing resistance worked out in Q3 and now the unknown value. The supply is still 6 V. Have fun!

5: Series and parallel circuits

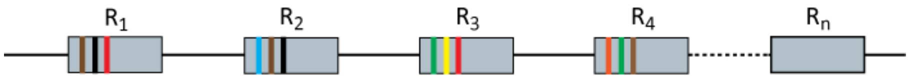
In this session the student will:

- Apply Ohm's law to series and parallel circuits.

Now that you are fairly comfortable with the idea of loads being connected to a source of supply, let us develop this more fully.

Series circuits

When more than one resistor or load is connected end to end, then the resistors are said to be connected in series.

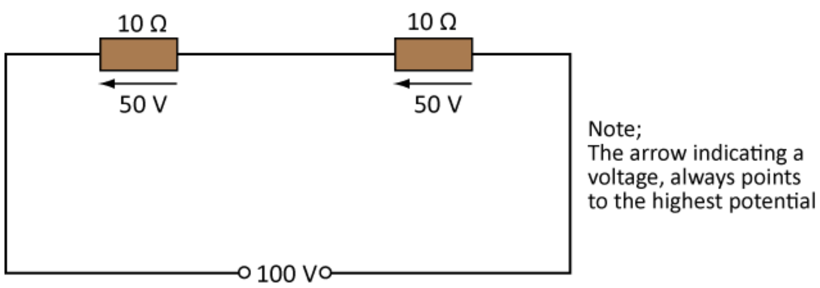


Resistors or loads

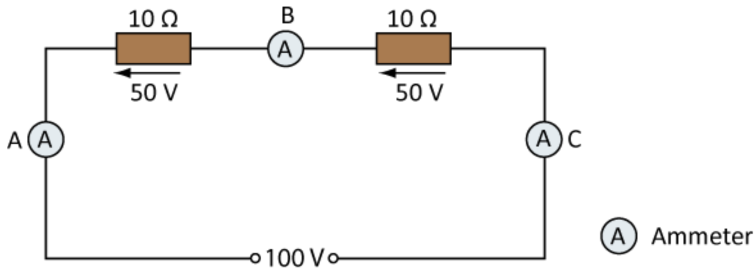
Let us see how the voltage and current are arranged and from this we should be able to develop the equation for finding the total resistance of the circuit.

Consider a simple circuit containing two resistors in series and connected to a supply source.

I will assume that the source or supply is 100 V and that the two resistors have the same value, say 10 Ω.



What we could say is that the voltage that is dropped across each resistor would be exactly half the total supplied, that is 50 V since the resistors are equal in value.



What we can also see is that the current must be the same at all points of the circuit. That is the current at point (**A**) is the same as at point (**B**) and at (**C**).

Looking at this more mathematically we can say that the total voltage supplied is the sum of the voltages dropped across the resistors, or $U_{Total} = U_1 + U_2$.

Now as you are so very familiar with Ohm's Law, you will be able to recognise that $U = IR$. What we can do is substitute IR for U every time we see it.

From this we get:

$$U_{Total} = U_1 + U_2$$

$$\bar{I}R_{Total} = \bar{I}R_1 + \bar{I}R_2 \quad \text{current cancels as it is constant}$$

$$R_{Total} = R_1 + R_2$$

$$R_T = R_1 + R_2 + ... + R_n$$

It may seem complicated but it is important for you to know that because the current is constant it can be cancelled and so things can be simplified.

The equation that you have to remember is the last one.

$$R_T = R_1 + R_2 + ... + R_n$$

The more resistors connected the longer the equation will become. $\mathbf{R_n}$ just stands for any number of resistors.

We'll look at a couple of examples.

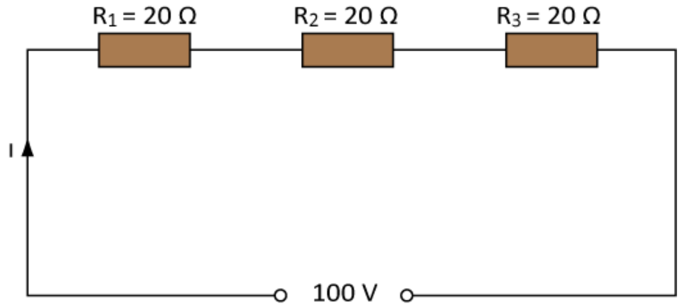
1). Three lamps are connected in series. They have resistance values of 20 Ω each. If the supply voltage is 230 V what will be:

i) total resistance

ii) total current

iii) voltage dropped across each resistor.

Always try to draw a diagram. It is invaluable in giving you an idea of what you are looking for. Remember it is a thumbnail sketch; do not spend too long on it.



There is a lot here so take your time in following it. The first section deals with adding up the resistors. The second section finds the current using our old friend Ohm's Law. The third section finds the voltages dropped across the loads, again using Ohm's Law.

Determine the total resistance first

i). $R_T = R_1 + R_2 + R_3$

$R_T = 20 + 20 + 20 = \underline{\underline{60\Omega}}$

From this we can now find the total current flow

ii). $U = IR$ transpose

$I = \frac{U}{R}$

$I = \frac{230}{60} = \underline{\underline{3.83A}}$

Current is the same in all parts of a series circuit so we can now determine the volt drop across each resistor

iii). $U_1 = IR_1 = 3.83 \times 20 = \underline{\underline{76.6V}}$

$U_2 = IR_2 = 3.83 \times 20 = \underline{\underline{76.6V}}$

$U_3 = IR_3 = 3.83 \times 20 = \underline{\underline{76.6V}}$

There are three effective questions here and it is important that you don't try to rush the answer. Show all your working and fill in your thumbnail sketch as you find out more.

This next example is similar to the one you attempted earlier,

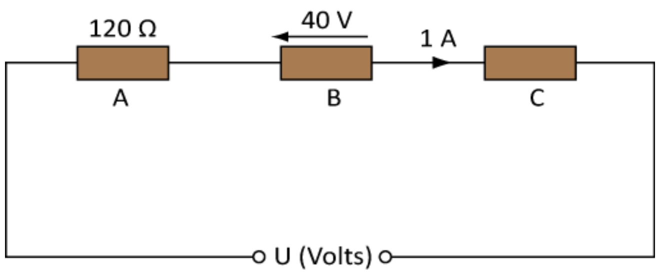
2). Three resistors are connected in series A, B, and C. A has a resistance of 120 Ω, B has a volt drop across it of 40 V when 1 A flows. The total resistance of the circuit is 300 Ω. Calculate:

i) resistance of B and C

ii) volt drop across A and C

iii) total volt drop.

Certain principles should have been remembered by now. Firstly, in a series circuit, the current is constant, and so the current through (B) is the same as for (A) and (C). The second principle is to draw a thumbnail sketch.



The working out is shown below.

i).

$$U = IR \quad \text{transpose}$$
$$R_B = \frac{U}{I}$$
$$R_B = \frac{40}{1} = \underline{\underline{40\Omega}}$$
$$R_T = 300\Omega$$
$$R_T = R_A + R_B + R_C \quad \text{transpose}$$
$$\therefore R_C = R_T - R_A - R_B$$
$$R_C = 300 - 120 - 40 = \underline{\underline{140\Omega}}$$

ii). In this instance because we now know the resistance of each of the resistors and the current in the circuit, all we have to do is apply Ohm's Law. Remember it is vitally important that you have the first part to the question correct. If you haven't, all the rest will be wrong.

$$U_A = IR_A = 1 \times 120 = \underline{\underline{120V}}$$
$$U_C = IR_C = 1 \times 140 = \underline{\underline{140V}}$$

iii). You can choose two ways in which to answer this last part.

- Either recognise that both the total current and total resistance are known, at which point $U = IR$, or,
- Add up the voltages that have been dropped across the loads.

Either way the answer will be the same. In fact, it is a check for you so that you can be sure that you have the right answer; the total supply voltage is always the sum of the other voltages.

$$U = IR$$
$$U = 1 \times 300 = \underline{\underline{300V}} \quad \text{or}$$
$$U_T = U_A + U_B + U_C$$
$$U_T = 120 + 40 + 140 = \underline{\underline{300V}}$$

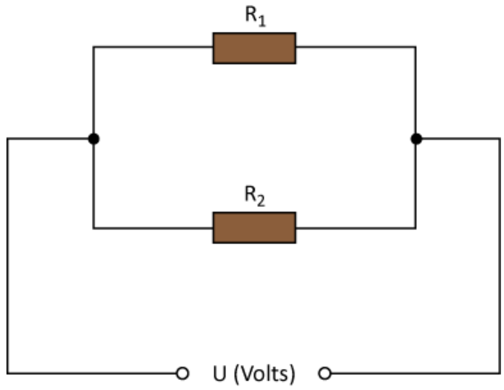
I know it seems to take a lot of time and paper to come to the right answer, but it is important that you take your time and follow the procedure. It cannot be repeated often enough:

In a series circuit, the current is constant and never changes. In addition, the total voltage is the sum of all other voltages dropped across the loads or resistors.

Parallel circuits

We have just looked at series circuits, and I hope that you can see that there is nothing to be frightened about when trying to work out any problems that come up. We are now going to look at parallel circuits.

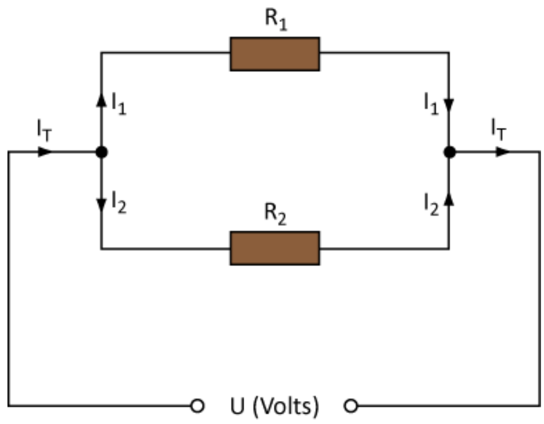
A parallel circuit is one where all the resistors have the same voltage across them.



You can see that the supply is connected to the ends of each resistor. The first important principle of parallel circuits can be seen here, and that is the voltage supplied to each resistor in a parallel network, irrespective of the size of resistor is the same.

What happens to the current in a parallel circuit? What you must remember is that the current flowing into a point must equal the current flowing away from a point.

This makes very good sense. Below we can see this principle at work. The current supplied comes to a joint or junction, there are only two ways for it to travel, and it must therefore divide down the two paths available to it.



In a parallel circuit, the voltage dropped across the loads is the same as the source voltage. It is the current that divides.

The total current supplied is the sum of the currents in each of the legs of the network, or $I_{\text{Total}} = I_1 + I_2$. This is opposite to what happens with a series circuit, if you remember the current is constant.

As with the series circuits we can substitute Ohm's Law, $I = \frac{U}{R}$ in the above equation. What we get is this:

$$I_{\text{T}} = I_1 + I_2$$
$$I = \frac{U}{R} \quad \text{substitute}$$
$$\frac{\bar{U}^1}{R_{\text{T}}} = \frac{\bar{U}^1}{R_1} + \frac{\bar{U}^1}{R_2}$$
$$\frac{1}{R_{\text{T}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Here, $\frac{U}{R}$ is substituted for I whenever it is seen. Because the voltage is constant it can be cancelled out and so U becomes 1.

The part of the equation to remember is the last line. The $\boldsymbol{R_n}$ means that this is true for any number of resistors in parallel.

$$\frac{1}{R_{\text{T}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

Some examples should help you to become familiar with the ideas and principles set out here.

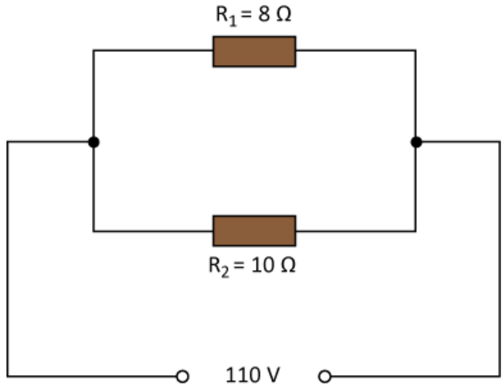
1). Resistors of 8 Ω and 10 Ω are connected in parallel. If the supply voltage is 110 V, calculate:

i) total resistance

ii) total current

iii) current in each resistor.

Remember to draw a small thumbnail sketch, with as much detail as possible on it.



i).

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{10}$$

$$\frac{1}{R_T} = 0.125 + 0.1 = \underline{\underline{0.225}}$$

$$R_T = \frac{1}{0.225} = \underline{\underline{4.44\Omega}}$$

Notice that you cannot just turn the equation upside down. Notice also that it is a question of doing fractions.

Another way of determining the total resistance of this parallel circuit could be to use the product over the sum rule:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{8 \times 10}{8 + 10} = \frac{80}{18} \therefore R_T = \underline{\underline{4.44\Omega}}$$

This only really works best when there are two resistors in parallel. That is not to say it cannot be used for multiple parallel connected resistors, it just become unwieldy. It would be best to use the reciprocal key on your calculator.

We are now looking for the total current.

ii).

$$U = IR \quad \text{transpose}$$

$$I = \frac{U}{R}$$

$$I = \frac{110}{4.44} = \underline{\underline{24.75A}}$$

Once the resistance has been found it is merely the application of Ohm's Law. Remember to fill in your thumbnail sketch with more detail as you find more out.

iii). In this part, what you have to remember is that the voltage is constant. All we need to do is apply Ohm's Law twice; once for each of the resistors.

$$U = IR \quad \text{transpose}$$

$$I = \frac{U}{R}$$

$$I_8 = \frac{110}{8} = \underline{\underline{13.75A}}$$

$$I_{10} = \frac{110}{10} = \underline{\underline{11A}}$$

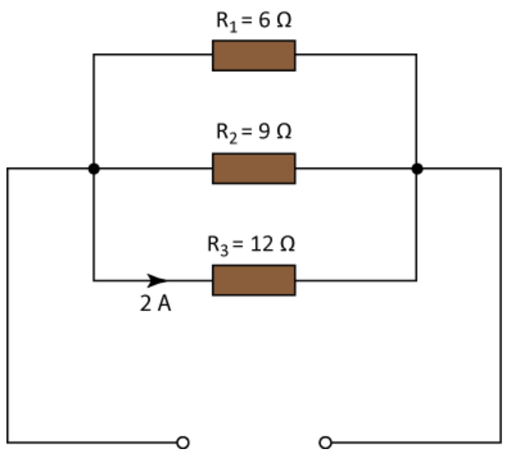
Notice that if you add up the two currents they equal the total current supplied, this is a useful check. If they don't add up then you have gone wrong somewhere so go back and check!

Another example will be helpful.

2). Three resistors of 6 Ω, 9 Ω and 12 Ω are connected in parallel. A current of 2 A is flowing in the 12 Ω. Calculate:

i) supply voltage
ii) current in each resistor leg
iii) total current
iv) total resistance

Remember to draw a thumbnail sketch.



Here what you have to remember is that the voltage dropped across any one resistor in the parallel network is the same for all resistors in the network. So because the 12 Ω resistor has 2 A flowing through it we can work out the voltage dropped across that resistor. This means we will have worked out the voltage across all three.

I will not explain everything this time; just see if you can follow the working out.

Find the total voltage

i). $U = IR$

$$U = 2 \times 12 = \underline{\underline{24V}}$$

Find the current in each leg

ii). $I = \frac{U}{R}$

$$I_6 = \frac{24}{6} = \underline{\underline{4A}}$$

$$I_9 = \frac{24}{9} = \underline{\underline{2.67A}}$$

Find the total current

iii). $I_T = \frac{U}{R}$ we don't know R yet

So $I_T = I_6 + I_9 + I_{12} = 2.67 + 4 + 2 = \underline{\underline{8.67A}}$

Find the total resistance

iv). $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{9} + \frac{1}{12}$$

$$\frac{1}{R_T} = 0.166 + 0.111 + 0.083 = \underline{\underline{0.361}}$$

$$R_T = \frac{1}{0.361} = \underline{\underline{2.77\Omega}}$$

I know that this seems very complex, but notice that Ohm's Law is very prominent once again. Take things step by step and don't worry too much about where you start. In many instances we can have different starting positions for the problem and the answers will still drop out OK.

We could calculate the total resistance by using the product over sum rule twice.

$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$R_T=\frac{R_1R_2}{R_1+R_2}=\frac{6\times 9}{6+9}=\frac{54}{15}=\underline{\underline{3.6\Omega}}$$

Now repeat for the remaining resistor

$$R_T=\frac{R_1R_2}{R_1+R_2}=\frac{3.6\times 12}{3.6+12}=\frac{43.2}{15.6}=\underline{\underline{2.77\Omega}}$$

Notice that the numbers come out the same.

Exercise 5.

1) If a lamp has a resistance of 960 Ω and a supply voltage of 230 V. What will its current be?

2) What remains constant in a series circuit?

3) Two resistors of 7 Ω and 9 Ω are connected in series. If the supply voltage is 230 V, what will be its total current and the voltage dropped across each resistor?

4) Two resistors of 7 Ω and 9 Ω are connected in parallel. If the supply voltage is 230 V, what will be the total resistance, the total current and the current in each leg?

5) State the difference between emf and volt drop.

6) Three resistors each of 8 Ω are connected:

   a) in series

   b) in parallel.

   Find out anything you can about each of the circuits, if they are supplied from a 110 V source.

7) A load draws a current of 220 A through a cable having a resistance of 0.03 Ω/100 m of single conductor. The length of run is 230 m.

   a). What will be the total resistance of the cable if a second cable of the same size is connected in parallel with the first?

   b). What will be the new voltage at the load if the supply voltage is 245 V?

8) Two loads, A and B draw currents of 25 A and 38 A respectively. The resistance of the two-wire cable is 0.12 Ω/100 m twin cable. The cable runs for 70 m from the distribution board to the first load and then a further 24 m to the next load. Determine:

   a). The current drawn in each leg of the circuit

   b). The resistance of each part of the circuit [Not the loads]

   c). The volt drops in each leg of the circuit.

6: Combined series and parallel circuits

In this session the student will:

- Apply Ohm's law to combined series and parallel circuits.

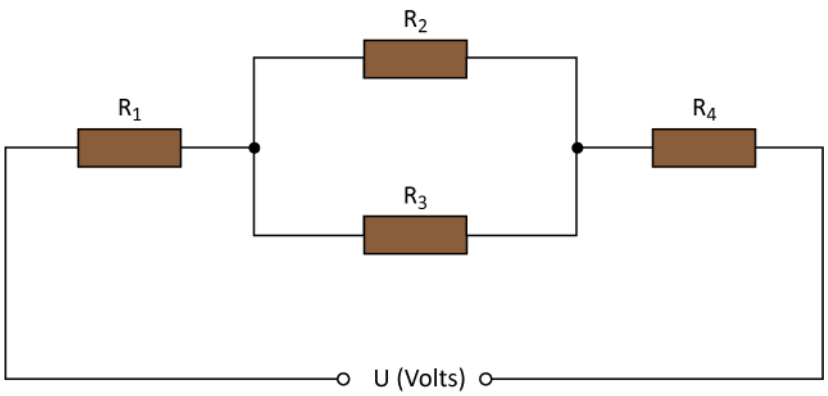
In this session on series and parallel circuits, we are going to spend some time considering what happens when we start to combine series with parallel circuits. In the last two sessions we have considered basic Ohm's law and resistance and have seen that resistance if dependent on the material, its length and area, and to an extent the temperature. We also have considered both series and parallel circuits as distinct circuit types.

The rule for a series circuit is that the **current is constant** and never changes. This makes sense as there should be no path for the current to flow other than the one laid out in the circuit.

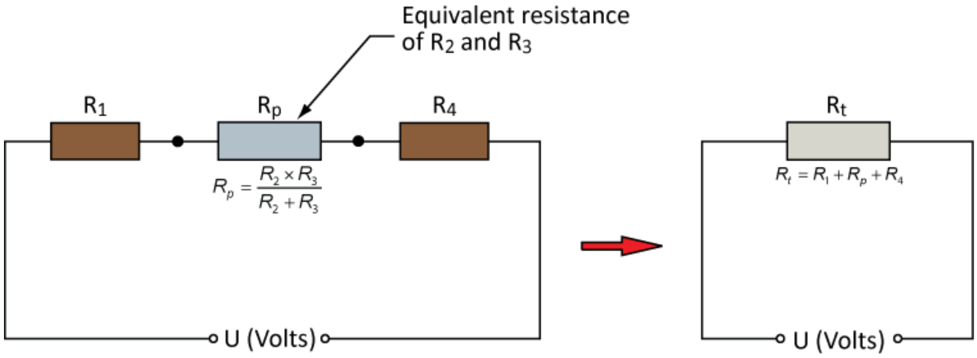
The rule for the parallel circuit is that the **voltage across any resistors is the same** for all resistors. We can take these rules and start to apply them to other combined circuits.

Series/parallel circuits

Series/Parallel arrangements are not complex; they are merely a combination of series circuits and parallel circuits.



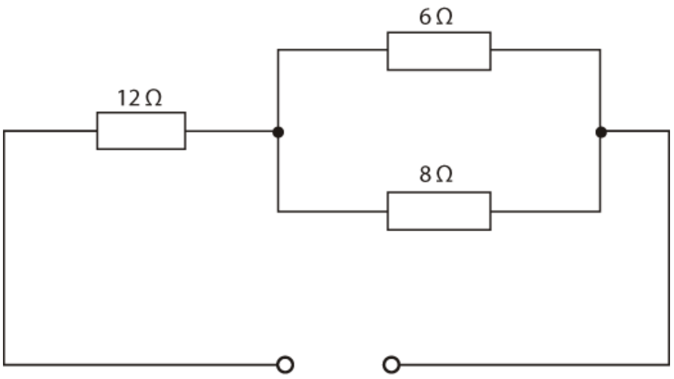
What you have to do is to deal with each of the parts of the circuit in turn. It is usually best to deal with the parallel part of the network first and reduce it to an effective single resistor. You can now see that this has become a simple series circuit.



Follow this example.

1). A circuit has three resistors of 6 Ω, 8 Ω and 12 Ω. The 6 Ω and 8 Ω resistors are connected in parallel with each other. An additional 12 Ω resistor is then connected in series with them. Calculate the total resistance.

Remember to draw your thumbnail sketch. Do the parallel part first.

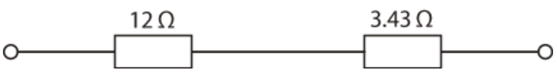


$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{8} = 0.166 + 0.125 = \underline{\underline{0.2917}}$$

$$R_T = \frac{1}{0.2917} = \underline{\underline{3.43\Omega}}$$

Draw another small sketch just to show what effective circuit you now have.



All we have left is a series circuit.

$$R_T = R_1 + R_2$$

$$R_T = 3.43 + 12 = \underline{\underline{15.43\Omega}}$$

Exercise 6.

1) What remains constant in a series circuit?

2) What remains constant in a parallel circuit?

3) If three lamps were connected in series and each had the same resistance, what would be the voltage dropped across each one if the supply was 230 V?

4) Three resistors of 4 Ω, 16 Ω and 24 Ω are connected in parallel. If the supply voltage is 110 V, what will be the:

   i) total resistance
   ii) current in each leg of the circuit
   iii) total current.

6) Four resistors are connected in a series/parallel circuit. The 6 Ω and 12Ω resistors are connected in parallel with each other and are then connected in series with the 17 Ω and 27 Ω resistors that are also connected in parallel with each other. If the supply voltage is 230 V calculate:

   i) total resistance
   ii) total current
   iii) voltage dropped across each network
   iv) current in each leg of the networks.

8) Two resistors of 5 Ω and 10 Ω are connected in parallel across a battery. The emf of the battery is 24 V and the internal resistance is 1.2 Ω. Determine the total current in the circuit and the terminal voltage of the battery. What will be the voltage dropped across the two parallel resistors?

   (All batteries have some resistance and this is called internal resistance).

7: Power

In this session the student will:

- Calculate the power dissipated in circuits.

Electrical energy

As with thermal and mechanical energy, electrical energy is measured in joules (J), and as with the other two forms of energy the work done is dependent on a number of elements. With thermal energy we have to consider the mass, specific heat capacity and the temperature of the substance. With mechanical energy we are dealing with mass, velocity, height and acceleration due to gravity depending on whether we are looking at kinetic or potential energy. With electrical energy the elements that we consider depend upon the flow of current and the voltage that is dropped across a particular component.

From an earlier study, electrical energy was found from the formula; $W = IUt$

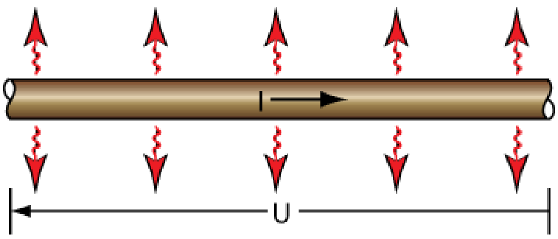
Where:

$W =$ electrical energy in J

$I =$ current in A

$U =$ voltage in V

$t =$ times in s



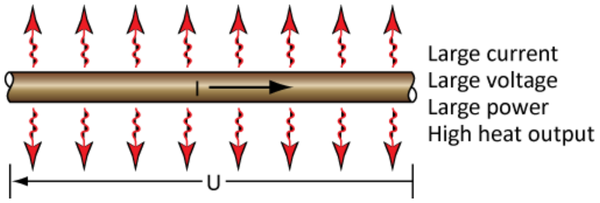
As the current flows heat is given off, with time the amount of energy consumed will increase.

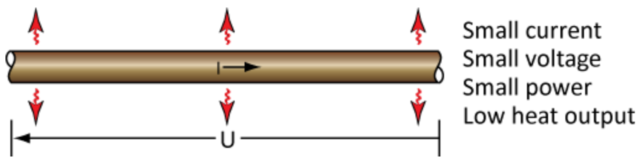
The heat given off from a cable or a load is not a good thing, it is wasted energy that the consumer pays for but does not get any useful work done from it. The better the conductor, the lower the resistance and the less heat emitted as waste. This heat can be dangerous if it is not allowed for in design calculations. Overheating cables can lead to fire risk.

Power

Power and energy have already been touched on in ***mechanics***. We did not look at them too closely knowing that we are more concerned with electrical power and energy.

In effect, with power we are more concerned with how quickly the heat is given off by the load.





The larger the voltage and current, the greater will be the power output and this can be demonstrated by:

$$P = IU$$

Where:

$P =$ power in W

$I =$ current in A

$U =$ voltage in V

Power, in whatever system we deal with is measured in watts (W) and:

$$1 \text{ watt} = 1 \text{ Joule/second} \left({}^{J}\!/_{s} \right)$$

1). Energy has already been defined as '***work done***' or just plain '***work***'.

2). Power has been defined as the '***rate at which work is done***'.

Both of these statements are true for electrical power and energy as well as for mechanical power and energy.

When current flows in a resistive circuit, heat is given off (energy released). The rate at which that heat is given off is determined by the power in the circuit.

The formula that describes the energy converted in an electrical circuit takes three forms. These three forms although they look very different, will give the same answer.

$$W = IUt \quad W = I^2Rt \quad W = \frac{U^2t}{R}$$

You can see that they all look so different. The common relationship between them all however, is Ohm's Law. It is not necessary to spend too much time looking at energy at present, but one or two extra details are worthwhile knowing.

In the three formulae above, the other common factor between all three equations is time. To determine the three most common equations for power, all that has to be removed is the time factor.

$$P = IU \quad P = I^2R \quad P = \frac{U^2}{R}$$

You can see how similar to the energy equations they are. These three equations are amongst the most important equations that you are likely to come across. It is critical that you can memorise them, and are able to transpose them.

On this page is a table that lists all the variations on Ohm's Law and the power formulae.

$U = IR$	$I = \frac{U}{R}$	$R = \frac{U}{I}$
$P = IU$	$I = \frac{P}{U}$	$U = \frac{P}{I}$
$P = I^2R$	$I = \sqrt{\frac{P}{R}}$	$R = \frac{P}{I^2}$
$P = \frac{U^2}{R}$	$U = \sqrt{PR}$	$R = \frac{U^2}{P}$

I hope that these are of use to you, although it would be better if you can try to remember the top row and the left-hand column. Try transposing them to get to the same as in the boxes. You will find help in transposing in the ***Maths*** study book; have a look back at it.

It would be worthwhile to try one or two examples, just to become familiar with the power formulae.

1). Calculate the current demanded by a 60 W, a 100 W, and a 150 W lamp when they are connected to a 230 V supply.

$$P = IU \quad \text{transposing}$$
$$I = \frac{P}{U}$$
$$I_{60} = \frac{60}{230} = \underline{\underline{0.26\text{A}}}$$
$$I_{100} = \frac{100}{230} = \underline{\underline{0.435\text{A}}}$$
$$I_{150} = \frac{150}{230} = \underline{\underline{0.652\text{A}}}$$

I have put all three into one here. It is essential that the transposing of the formula comes first. From then on, all you have to do is fill in the numbers because you are following the same process three times.

Try this next one.

2). The insulation resistance between two conductors is 0.5 MΩ. The p.d. between the two conductors is 200 V, what will be:

a) the leakage current

b) the power loss.

$$I = \frac{U}{R}$$
$$I = \frac{200}{0.5 \times 10^6}$$
$$I = \underline{\underline{400\mu\text{A}}}$$

Two steps are necessary here. The first is to work out the current, and here we are back to the old tried and trusted Ohm's Law.

The answer from this first part can then be used for the second part, although this is not necessary.

In this case I will use the three power formulae, just to show that it doesn't matter which one is used, as they all give the same answer.

$$P = IU$$
$$P = 400 \times 10^{-6} \times 200 = \underline{\underline{0.08\text{W}}}$$
$$P = I^2R$$
$$P = \left(400 \times 10^{-6}\right)^2 \times \left(0.5 \times 10^{-6}\right) = \underline{\underline{0.08\text{W}}}$$
$$P = \frac{U^2}{R}$$
$$P = \frac{200^2}{0.5 \times 10^{-6}} = \underline{\underline{0.08\text{W}}}$$

One more example should be enough.

3). A cooker circuit is fed from a 32 A miniature circuit breaker, with a supply voltage of 230 V. What will be the maximum power consumed by the cooker?

$$P_{\text{cooker}} = IU$$
$$P_{\text{cooker}} = 32 \times 230 = \underline{\underline{7360\text{W}}}$$

Try the exercise on the next page. They contain harder questions which you will need to give some thought to. Draw a sketch, enter the information you know, think about the formula's you will need, look back at your notes for clarification. As you move through your electrical course the questions will contain elements you studied earlier.

Exercise 7.

1) If a lamp has a resistance of 960 Ω and a supply voltage of 230 V what will its power rating?

2) Two resistors of 7 Ω and 9 Ω are connected in series. If the supply voltage is 230 V, what will be the total power dissipated and the power dissipated from each resistor?

3) A motor draws 60 A from a distribution point 140 m away. The cable has a resistance of 0.01 Ω/m per 100 m of single conductor. The voltage at the intake position is 425 V. What will be the power dissipated in the load and in the cables?

4) An aluminium cable, 350 m long, has two aluminium conductors, each having a cross-sectional area of 95 mm^2. A current of 124 A is drawn by the supply. Calculate:

   a) Voltage drop in the cable if the resistivity is assumed to be $2.83 \times 10^{-8}\,\Omega m$

   b) The voltage at the load if the supply is 245 V

   c) The power dissipated in the cable, the total power available, and the power available at the load.

8: Effects of an electric current

In this session the student will:

- Gain an understanding of the effects of an electric current.

There are three effects of an electric current. These are '***chemical***', '***magnetic***' and '***heating***'.

Chemical effects of an electric current

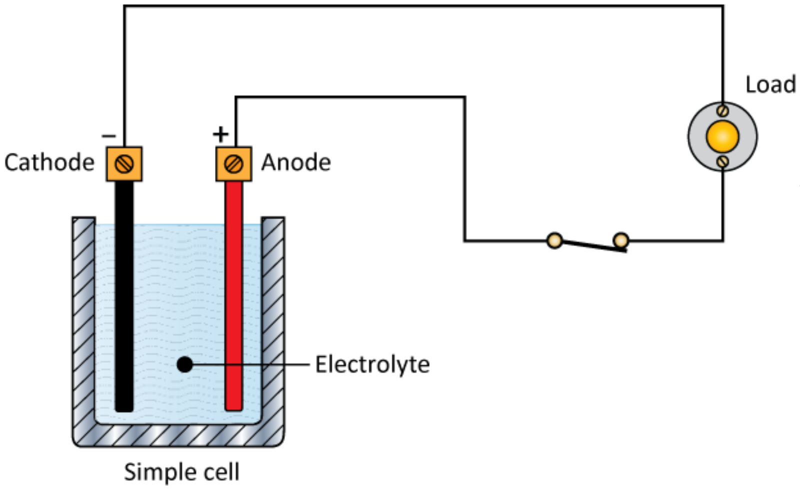
In the electrical trade, whenever chemical is mentioned we always think of batteries. This is no exception.

There are two types of batteries: primary batteries (disposable batteries), which are designed to be used once and discarded, and secondary batteries (rechargeable batteries), which are designed to be recharged and used multiple times. Batteries come in many sizes; from miniature cells used to power hearing aids and wristwatches to battery banks the size of rooms that provide standby power for telephone exchanges and computer data centers.

Secondary battery

Primary battery

A battery is made up of cells; the diagram below shows a single cell in its simplest form. Whether it is a primary or a secondary cell depends upon the electrolyte used. The electrolyte needn't be a liquid; it can just as easily be a paste.



The anode is always the positive terminal of a cell or battery and the cathode is the negative. These terms will be revisited when you study the electronics unit.

When electric current passes through an electrolyte it '***breaks down***' and hydrogen ions (positive) are collected at the cathode. Sulphate ions (negative) are collected at the anode; the anode and cathode merely being the two dissimilar metals. The passing of current through an electrolyte is called '***charging***'.

If a load is connected between the plates then current will flow from the anode to the cathode. Chemically, the opposite occurs to what has just occurred with charging. The delivery of current to a load is called '***discharging***'.

The energy supplied from a battery may be used to supply a load but will only be sufficient for a limited time, as the chemical qualities of the electrolyte deteriorate with use. Secondary cells cannot be re-charged indefinitely, after a number of charges and discharges the secondary cells become unable to hold their charge and the battery is finished. Primary cells become finished as soon as they lose their stored energy.

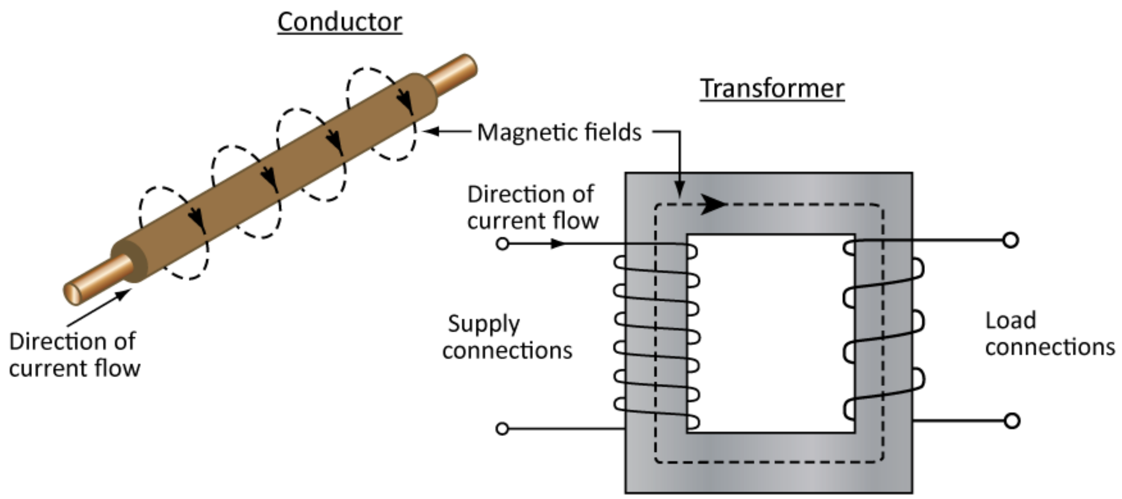
The source of energy from a battery, irrespective of whether it is a primary or secondary type, is termed an emf and stands for electromotive force. Incidentally, a solar cell or photodiode is another source of emf, with light energy as the external power source.

To increase the voltage the cells would be connected in series. To increase the current, the cells would be connected in parallel.

Magnetic effects of an electric current

Whenever an electric current passes through a conductor a ***magnetic field*** appears around it.

This magnetic field can be increased when the wire is formed into a ***coil***. If the coil is then wound on an iron core then the iron will become magnetized. Examples of this effect are numerous and are critical to the understanding of electrical principles.



More details will be given in a later session, but examples of the use of magnetism would be in speakers, motors, generators, doorbells, transformers, relays etc. The list is almost endless.

It is sufficient to know that an electric current passing through a conductor will produce a magnetic field around itself!

Heating effects of an electric current

Why is heat produced when current is passed through a wire?

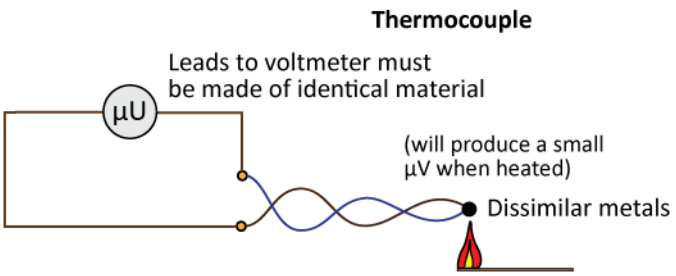
To answer this you need to think back to the session on electron theory.

A metallic conductor has a large number of free electrons in it. When a potential difference is applied across the ends of a metallic wire, the free electrons begin to drift from a region of low potential to a region of high potential. These electrons collide with the positive ions (the atoms which have lost their electrons). In these collisions, the energy of the electron is transferred to the positive ions and they begin to vibrate more violently. As a result, heat is produced. The greater the number of electrons flowing per second, the greater will be the rate of collisions and so greater is the heat produced.

The heating effect of current is utilised in the electrical heating appliances such as electric iron, room heaters, water heaters, etc. All these heating appliances contain coils of high resistance wire made of nichrome alloy. When these appliances are connected to a power supply by insulated copper wires then a large amount of heat is produced in the heating coils because they have high resistance, but a negligible heat is produced in the connecting wires because the wires have low resistance.

The heating effect of an electric current is utilized in electric lamps for producing light. When electric current passes through a thin high resistance tungsten filament of an electric lamp, the filament becomes white hot and emits light. This is why they are called incandescent lamps.

An 'electric fuse' is an important application of the heating effect of current. When the current drawn in a domestic electric circuit increases beyond a certain value, the fuse wire gets over heated, melts and breaks the circuit. This prevents fire and damage to various electrical appliances.



Another aspect to heat and current flow is shown by the ***thermocouple***. When two different metals are joined in a closed circuit and a temperature difference occurs then a voltage is produced.

This is called the '***thermoelectric effect***' and is very useful when temperature difference is trying to be determined in very hot areas.

Exercise 8.

1) Give three effects of an electric current.

2) Give two suitable examples of each of the effects stated in (1).

3) Give two types of cell, and where they might be used.

4) What is an electrolyte?

5) When would you use a thermocouple?

7) What is a combination of cells called?

8) How would you connect single 2 V cells to make a 12 V car battery?

9) How would you connect single 2 V cells for a load which requires high current?

Appendix 1

There are a series of practical tasks that you should now have sufficient knowledge to be capable of attempting. Suggested experiments/practical work sessions would include:-

1) Using a simple relay, normally open switches, normally closed switches, sounder and an appropriate d.c. supply, wire up and test an open circuit alarm and a closed circuit alarm.

2) Using resistors of various values, wire up a series and a parallel circuit to an appropriate d.c. supply.

   Take measurements of current and voltage at various points in the circuit/s and check whether the theory matches the practice. Take care to note that if the same resistors are connected in parallel as you have used in series then the current drawn from the supply will increase and care needs to be taken to match the power rating of the resistors to the current drawn.

3) Using at least three different resistors, vary the supply voltage from 0 V to no more than 10 V and measure the current flow. Draw a graph and check to see if Ohm's law is true.

These are just some suggested areas for practical activities that would be appropriate for you to understand.

B&B Training Associates

Engineering Learning Materials

Attempt all questions.

All marks are shown in the right-hand margin.

You should aim to pass with a 85 % minimum mark.

Anything less than this mark should lead you to re-read the text.

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1). What is it that is occurring when current flows? 2

2). In which direction does conventional and actual current flow? 2

3). If 13 A flows for 2 hours in a circuit what is the quantity of electricity that has passed? 3

4). What three effects are there of an electric current? 3

5). What is an electrolyte, where would you find it, and what does it do? 3

6). What is the difference between a primary and a secondary cell? 2

7). A coil has a resistance of 470 Ω. What will be the current that flows when a 24 V supply is connected? 3

8). A coil of 470 Ω has a resistor of 1 kΩ connected in series with it. If the supply is 24 V, determine: 6

- the total resistance
- the current
- the voltage dropped across each resistor.

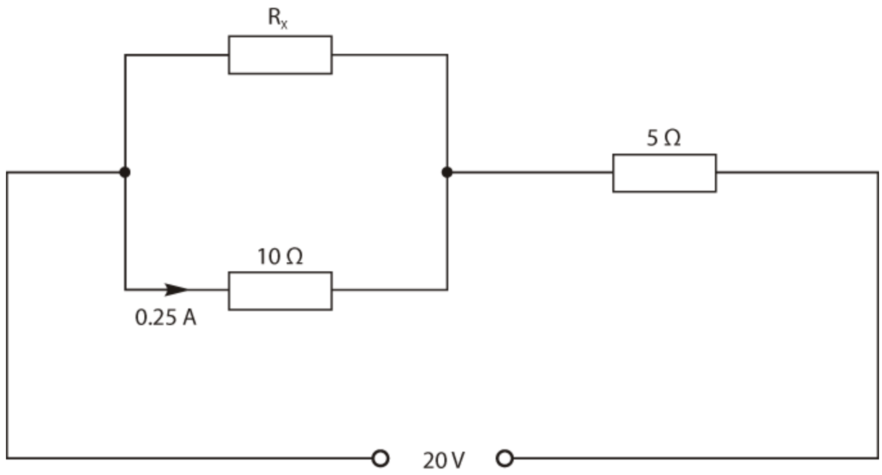
9). If the same resistors in question (9) are connected in parallel with the same supply, determine: 6

- the total resistance
- the total current
- the current in each resistor.

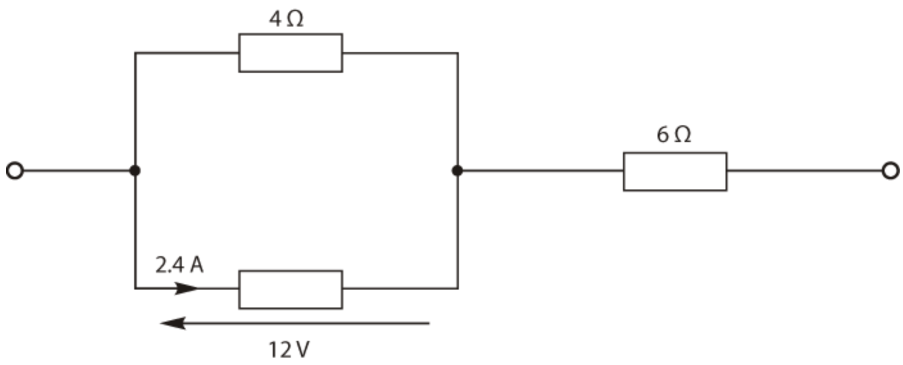
10). What remains constant in a series and in a parallel circuit? 2

11). If the area of a conductor halves and the length of it doubles, what will happen to the resistance of the conductor? 2

12). For the circuit shown below determine the value of resistor $\mathbf{R_x}$ to two (**2**) significant figures. 8



13). Determine the volt-drop across the 6 Ω resistor in the circuit below. 8



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