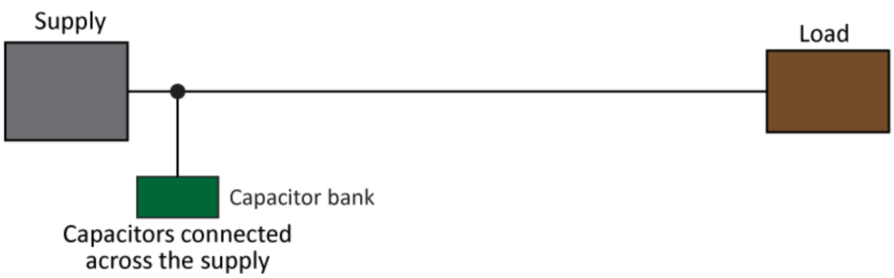
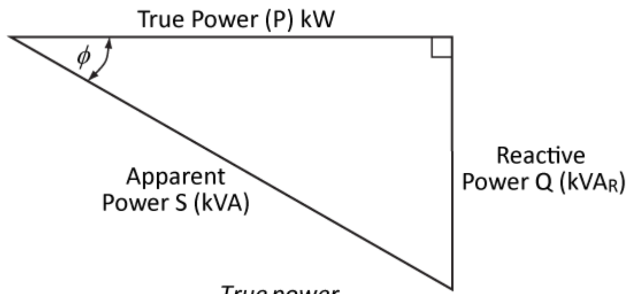
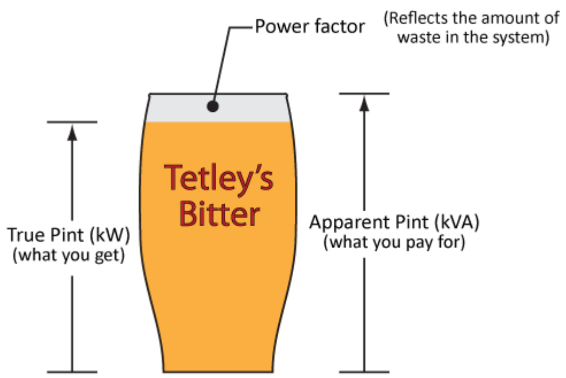
Level 3 Diploma
in
Installing Electrotechnical Systems & Equipment

C&G 2357

Unit 309-7B Understand how different electrical poperties can effect electrical circuits, systems and equipment







$$Power\ factor = \frac{True\ power}{Apparent\ power}$$

$$p.f. = \frac{P}{S}$$

$$\cos\phi = p.f$$

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Learning Materials for the Electrical Industry

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Aims and objectives

By the end of this study book you will have be able to:

- Explain the relationship between resistance, inductance, capacitance and impedance.
- Calculate unknown values of resistance, inductance, inductive reactance, capacitance, capacitive reactance and impedance.
- Explain the relationship between kW, kVAr, kVA and power factor.
- Calculate power factor.
- Explain what is meant by power factor correction and load balancing (neutral current).
- Specify methods of power factor correction.
- Determine the neutral current in a three phase and neutral supply.
- Calculate values of voltage and current in star and delta connected systems.

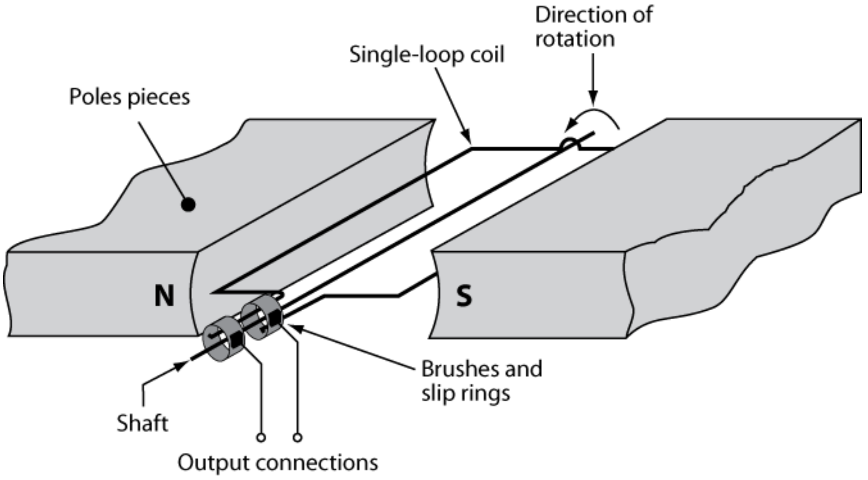
1: A.c. circuits

In this session the student will:

- Gain an understanding of how a.c. power is produced.
- Gain an understanding of what makes a.c. different from d.c.
- Use basic trigonometry for a.c. calculations and understanding.

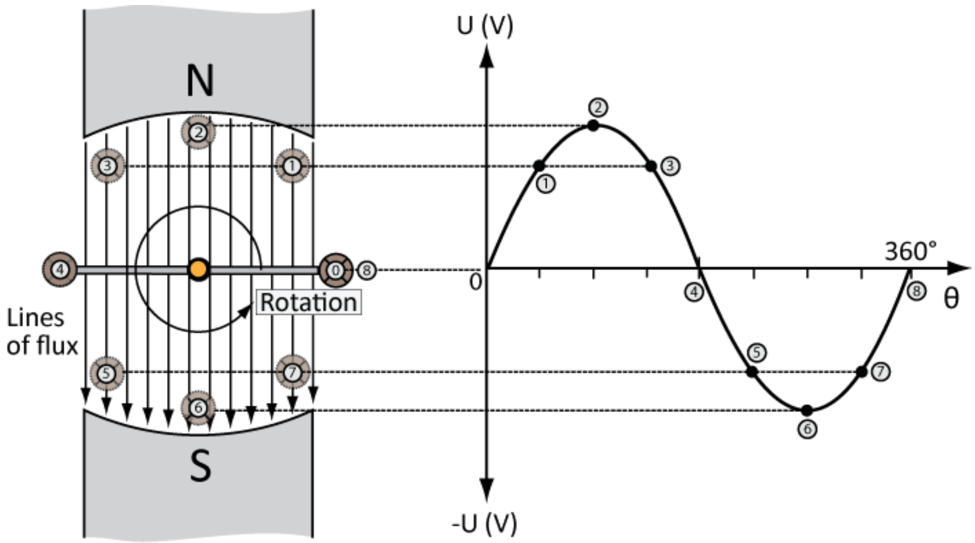
This session is predominantly revision. Up until now we have been discussing what happens when resistors, inductors and capacitors are connected to a d.c. source. In this unit we look at the effect an a.c. supply has upon the same components, you will find that their behaviour is quite different except for the resistor. There is quite a high mathematical content within this unit, but this will be explained as we go along.

You have already learnt that alternating current or a.c. is the supply most common in the UK; this is produced when a coil is placed within a magnetic field and allowed to rotate.



You can see that the coil is free to rotate within the magnetic field. As the coil turns, current is induced in the coil and this is '***tapped off***' at the '***slip rings***'. As the coil rotates the current in the coil varies depending on how much of the magnetic field is being '***cut***'.

Instead of using loads of separate drawings, just notice how the sine wave is developed as a coil moves through the magnetic field set up between the poles. Remember, the maximum value of voltage is generated when the coil is directly under the North Pole as most flux is being cut at this position. At positions 0, 4 and 8 the coil is moving parallel to the field and therefore no flux is being cut so no voltage is being generated.



Considering the diagram above, it moves from zero up to a maximum in one direction. It then moves from the maximum, back through zero and then on to a maximum in the opposite direction, and then to zero. When a coil has completed one of these sequences, it is then ready to begin another. Each complete sine wave is called a '***cycle***' or '***period***'. The quantity of cycles in one second is called the '***frequency***'.

A simple formula is attached to this idea.

$$T = \frac{1}{f} \quad \text{or can be expressed as} \quad f = \frac{1}{T}$$

where:- T = time (s)

f = frequency (Hz)

Each cycle or period lasts for a certain amount of time. This is usually measured in seconds or milli-seconds.

Follow these examples.

1). T = 20 ms. Calculate the frequency.

2). F = 1 kHz. What is the period?

3). F = 1 MHz. What is the period?

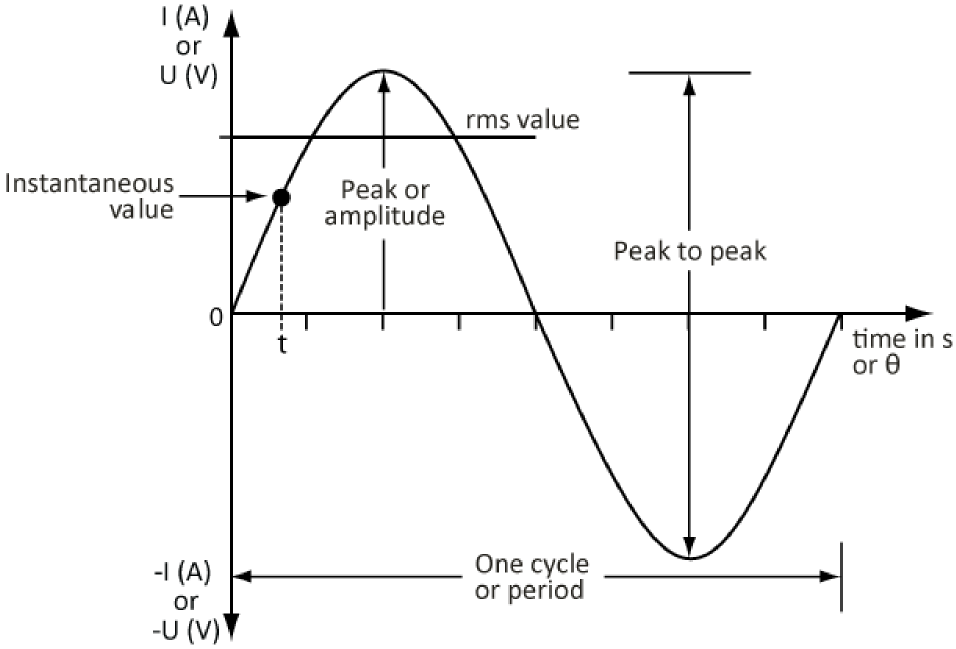
1). $f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = \underline{\underline{50\text{Hz}}}$

2). $T = \frac{1}{f} = \frac{1}{1000} = \underline{\underline{1\text{ms}}}$

3). $T = \frac{1}{f} = \frac{1}{1000000} = \underline{\underline{1\mu\text{s}}}$

Each part of the cycle above the zero line is called the positive half-cycle, and each part under the zero line is called the negative half-cycle.

A number of labels are attached to the sine wave. Look at the complete sine wave again.



From the centre or zero line to the peak of one of the half-cycles is called appropriately, the '***peak***' value. From the peak of the positive half-cycle to the peak of the negative half-cycle we have the '***peak-to-peak***' value.

We can see from the shape of the wave that it is not semi-circular, so the area under the wave is more complex to work out. Problems arise from that.

The maximum or peak value cannot be the total useful current, power or voltage, as so much of the wave is less than the maximum. This being the case another figure, something less than the peak, needs to be used.

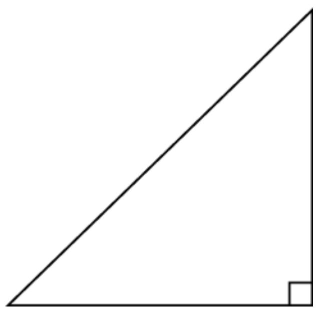
From the unit on magnetism, the figure used to describe the average value of voltage or current was 0.637 times the peak value. This means that if we had a peak value of voltage of 230 V, the average value would be 146.5 V. If the peak value of current was 10 A, the average value would be 6.37 A.

The average value does not give sufficient detail in a.c. work when most of the time we are dealing with power consumption. Because of this, we have to look at the 'heating effect' of the supply. This heating effect was also discussed in the unit on magnetism where you learnt that the rms value is always 0.707 times the maximum value.
This means that if we had a peak value of voltage of 325.4 V, the rms value would be 230 V. If the peak value of current was 10 A, the value would be 7.07 A.

One of the most important tools used with a.c. work is trigonometry. Whilst this has been covered before in the maths unit, that was a long time ago and a recap would be beneficial!

Trigonometry

We will first need to look at what is called a **right-angled triangle**. Have a look below.

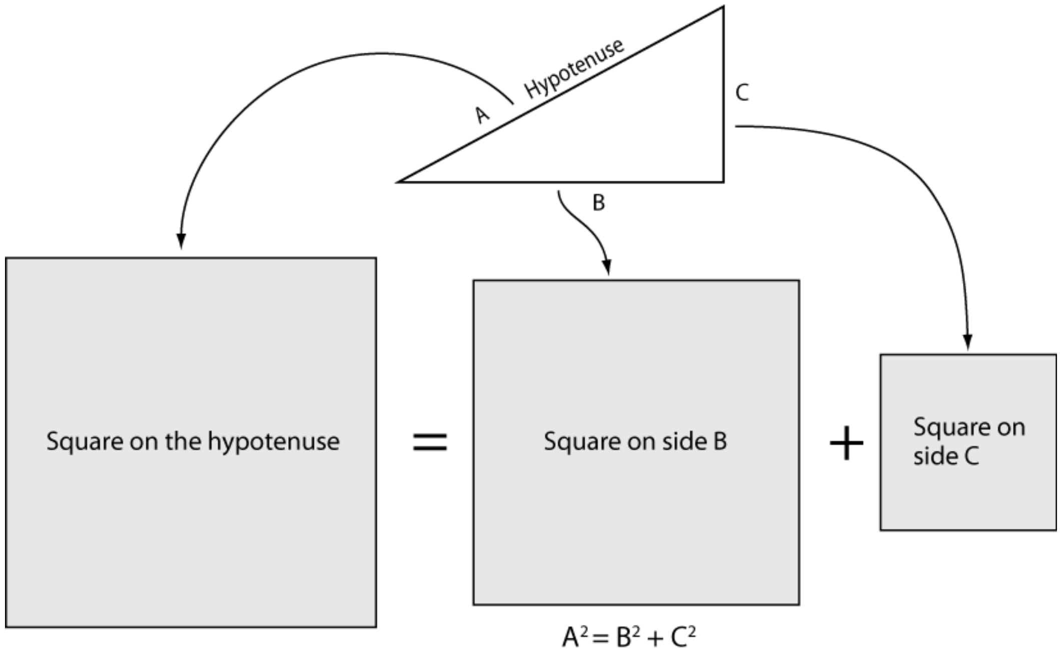


Triangles have a number of qualities. For example, all the angles add up to 180°. The extra qualities of right-angled triangles are of benefit to us in the electrical industry.

A Greek called Pythagoras stated:

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

So, what does this mean? Have a look below.



Here you can see that I have drawn a square on to each of the sides of the triangle. If you were to measure the individual areas, you would find that the two areas on the shorter sides of the triangle added up to the area on the longest side. The longest side is called the '***hypotenuse***'.

This relationship can and is described using a formula.

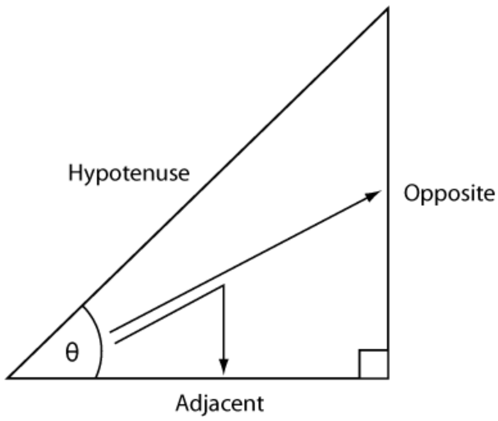
$$a^2 = b^2 + c^2$$

This can be transposed to form a slight variant.

$$a^2 = b^2 + c^2 \quad \text{transpose}$$
$$a = \sqrt{b^2 + c^2}$$

I have simply square-rooted both sides.

The second set of qualities that we can make use of is the relationships between the sides and the angles of a right-angled triangle.



In any right-angled triangle, we can describe any angle in terms of a ratio of two other sides.

Using the diagram above, assume that we are looking for the angle, θ (theta). We can find out this angle in any one of three ways.

It is here that we now have to introduce three terms that you may not have come across before, these are:

- **tangent** (tan);
- **cosine** (cos);
- **sine** (sin).

You don't need to worry about these new terms, although you will come to use them quite regularly. Have a look at the three formulae below.

1.
$$\text{sine } \theta = \frac{\text{opposite side of angle}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{O}}{\text{H}}$$

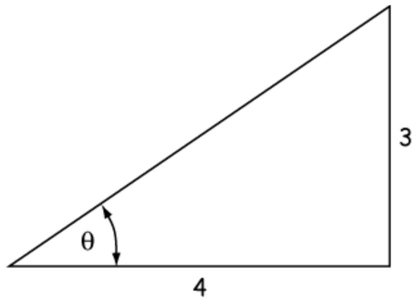
2.
$$\text{cosine } \theta = \frac{\text{adjacent side of angle}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{A}}{\text{H}}$$

3.
$$\text{tangent } \theta = \frac{\text{opposite side of angle}}{\text{adjacent side of angle}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{O}}{\text{A}}$$

Each of the new terms is a relationship between two sides of a right angled triangle.

It would be worthwhile just working through an example to see how it all relates.

1). Based on this triangle, determine the value of the angle θ and the unknown side. Use Pythagoras to aid the working out.



The length of the unknown side; $a = \sqrt{b^2 + c^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = \underline{\underline{5}}$

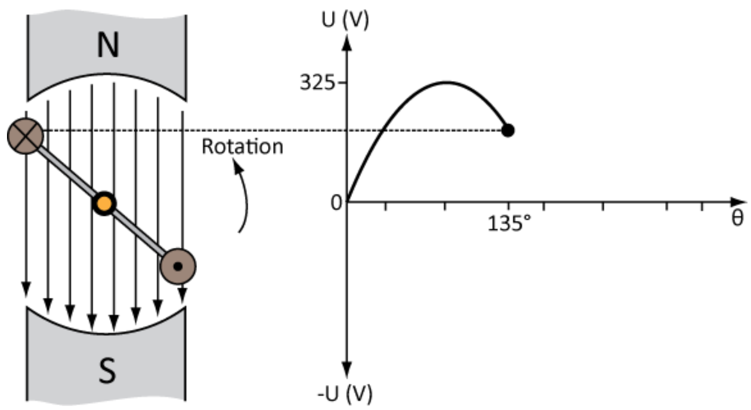
The angle θ using the cosine ratios; $\cos\theta = \frac{A}{H} = \frac{4}{5} = \underline{\underline{0.8}}$ *so* $\theta = \cos^{-1} 0.8 = \underline{\underline{36.86°}}$

The angle θ using the sine ratios; $\sin\theta = \frac{O}{H} = \frac{3}{5} = \underline{\underline{0.6}}$ *so* $\theta = \sin^{-1} 0.6 = \underline{\underline{36.86°}}$

A few points need to be noticed. Firstly, it doesn't matter which relationship is used. I can use ***sin, cos*** or ***tan***. The angles will still come out to the same value.

The second point is that the terms ***sin^{-1}*** or ***cos^{-1}*** or even ***tan^{-1}*** all have the same purpose, i.e. to turn the ratios of the sides into real angles.

Let's take our newfound skills and use them to find an instantaneous value of voltage at 135°.



The hypotenuse is the maximum value given here as being 325 V. What is the value of voltage when the coil is at his position?

Using the sine ratio;

$\sin 135 = \frac{x}{325} \therefore x = 325 \times \sin 135 \simeq 230V$

What do you notice about this value?

I hope you noticed that 230 V is the rms value of the supply and is the single-phase voltage we use today.

Exercise 1.

1) What is the period if the frequency is 75 Hz?

2) If the cycle is 7.6 ms, what will be the frequency?

3) What maximum voltage do I need to achieve an rms voltage of 254 V?

4) Determine the angles for the following cosine values:
   a) 0.57
   b) 0.95

5) Determine the angles for the following sine values:
   a) 0.57
   b) 0.95

6) For a rotating coil generating a maximum value of 325 V, determine the instantaneous voltage when the coil reaches the following positions.
   a) 30°
   b) 180°
   c) 270°
   d) 315°

7) What positions will the coil be in when the following voltages are being measured, if the maximum voltage is 500 V? Remember, there will be two positions which will give each voltage.
   a) 250 V
   b) 433 V
   c) -250 V
   d) -433 V

8) a) What will be the value of voltage at a time of 8 ms from switching on the supply if the rms value of supply voltage is 176.75 V at a frequency of 50 Hz?
   b) At what time from switching on would a voltmeter record the first -147 V value?

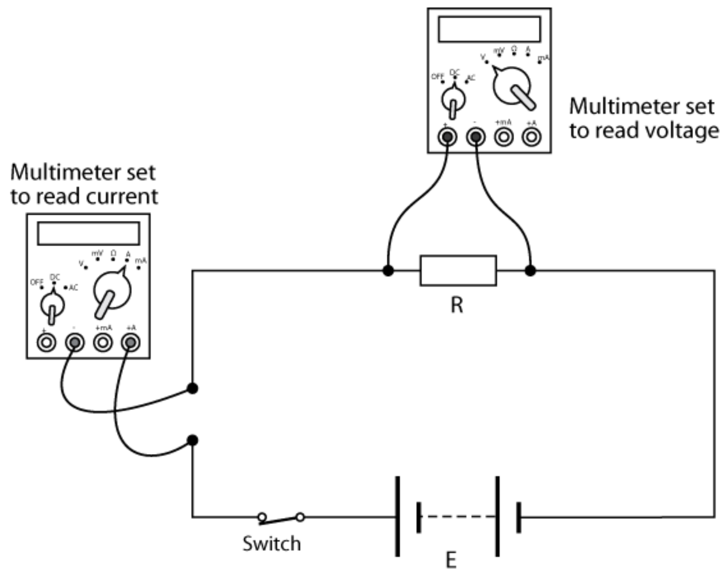
2: Effects of a resistor in an a.c. circuit

In this session the student will:

- Describe the effect a resistor has on an a.c. circuit.
- Gain an understanding of phasor diagrams.

In the first session we saw how a basic a.c. supply is generated. In this session we will start to consider what happens when we add various components into an a.c. circuit. In this session we will consider what happens when a resistor is added into the circuit.

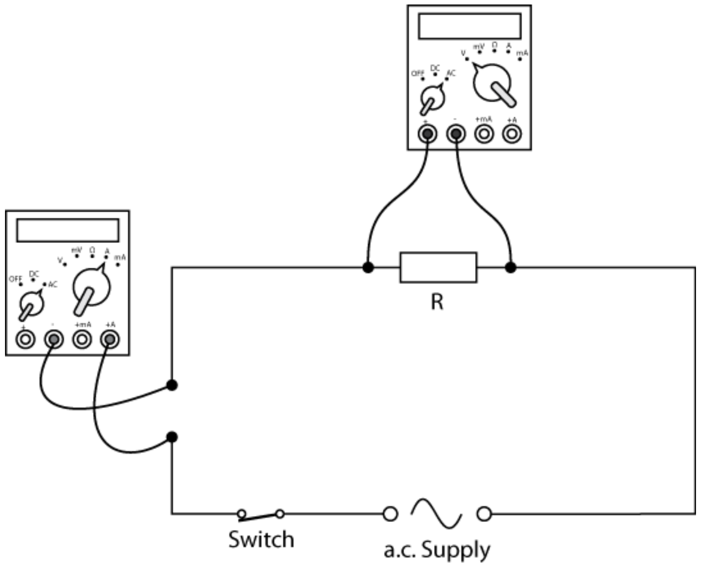
Consider this simple resistive circuit connected to a d.c. supply.



Here we have a resistor or load connected to a d.c. supply via a switch. If we have an ammeter connected in the circuit and a voltmeter connected across the load, then we would be able to determine the resistance of the load.

You are all aware of Ohm's law $U = IR$ and, the circuit above would not hold any problems for you. Now let's look at the a.c. supply.

Consider the diagram below.



The resistor shown here represents loads such as a heating element or a filament lamp etc. It is something that gets hot when a current is passed through it.

It is a similar diagram, but this time I have connected the load to an a.c. supply. The first thing that you should notice is the slightly different way that the symbol for the supply looks.

From your previous work, you will remember that we are always looking at the rms value as far as a.c. is concerned, and not the peak or maximum value. You should also remember that the rms value is a relationship between a.c. and d.c. The rms value is the equivalent d.c. heating effect. So if there is a 230V d.c. supply then this value can be related to a 230V rms. a.c. supply.

Therefore, what is the difference between a d.c. and an a.c. supply in its effect on this circuit?

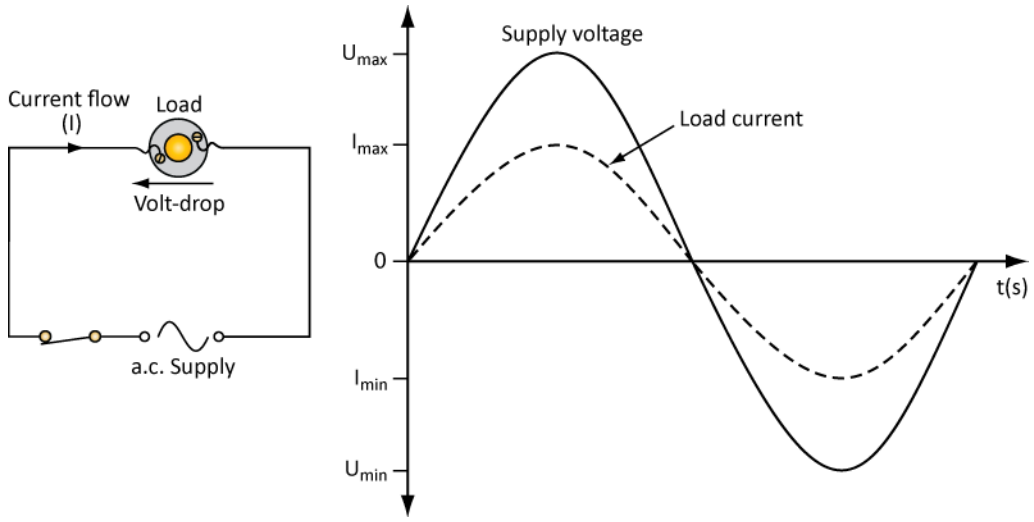
Nothing!

In a d.c. circuit the resistor, assuming that it is a pure resistor (without inductance), acts to ***oppose the current flow***. It acts a bit like a pinch on a hosepipe, which stops or slows the flow of water through the pipe.

With an a.c. circuit the same pure resistor acts in exactly the same way. Therefore there is no difference!

The next step in thinking that you have to make however is that of differentiating between what is happening to the current and what is happening to the voltage in this a.c. circuit.
Now because the a.c. supply is produced from a rotating alternator then the voltage and the current have both a quantity (how large is the value) and direction (in which sense is it acting).

Because they have quantity and direction, then they can be said to be **vector** quantities. Have a look below.



This waveform shows the current and the voltage superimposed on the same diagram.

You can see that as time passes then the voltage and current get either bigger or smaller, depending on where they are in their cycle.

Here the voltage and current depend on the size of the alternator, the number of windings, and how they have been connected, and the speed of rotation of the rotor. The direction is continually changing because the rotor is continually turning. This is of course why we get an a.c. supply.

A.c. is a vector quantity, we can draw any values that we have in the form of a vector diagram. In electrical terms, we call a vector diagram a ***phasor*** diagram.

Here the voltage rises and falls with the current following the same path, although it is commonly at a different value or quantity. The current is at zero at the same time as the voltage and it is at a maximum and minimum at the same time as the voltage. They rise and fall at the same time.

While a sine wave is useful as a picture to show what is happening, it is very time consuming to draw. A much better graphical representation is the phasor diagram.

Phasor diagrams.

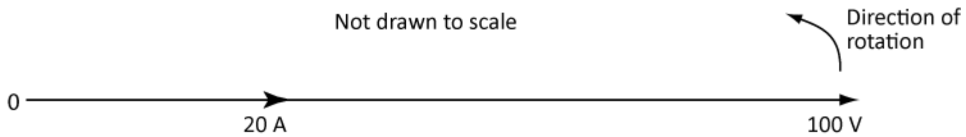
At this point it would be useful to mention the general rules for drawing phasor diagrams.

1. The length of a phasor is directly proportional to the value of the wave being depicted.
2. For series circuits, the supply current is drawn in the horizontal reference position as the supply current is common to all series connected components.
3. For parallel circuits, the supply voltage is drawn in the horizontal reference position as the supply voltage is common to all components connected.
4. The direction of rotation of all phasors is taken to be **anticlockwise.**
5. In any one phasor diagram, the same type of value such as rms, peak etc, is used and **not** a mixture of values.

The phasor diagram of the simple resistor circuit is shown below.

For this circuit it doesn’t matter whether the current or the voltage is in the reference position as there is only the one component. What is important is to show that the current is in phase with the voltage. This means that the current follows the voltage at each stage on the voltage waveform.

Assume the supply voltage is 100 V and the resistive load is 5 Ω, the circuit current will be 20A.



This may not appear to help us very much at the moment, but more will become apparent as we move on and look at more complicated circuits.

Exercise 2.

1) An a.c. voltage of 110 V feeds a load of resistance 65 Ω. What will be the current flow?

2) An a.c. circuit draws a current of 13.04 A when a 230 V supply is connected. What value of resistance does the heater have?

3) A resistive a.c. circuit fed from a 220 V supply draws 35 A of current. Draw a phasor diagram to scale of the elements.

3: Effects of an inductor in an a.c. circuit

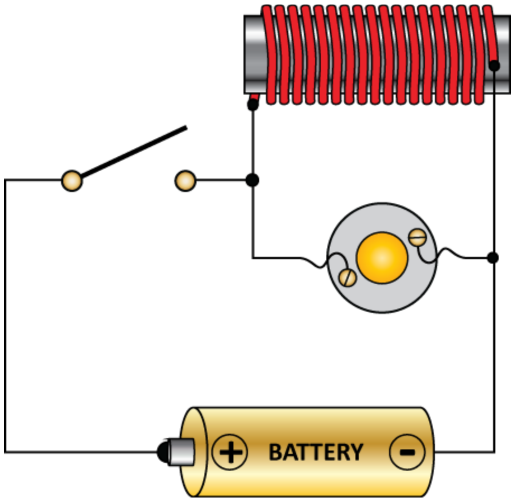
In this session the student will:

- Describe the effect an inductor has on an a.c. circuit.

We have already looked at inductance in a d.c. circuit. Inductance occurs when a length of wire is wrapped into a coil and the magnetic field around the coil increases as the current flows.

The magnetic field that is produced changes for a short period while it builds up, and this change leads to an emf being induced in the coil. This induced emf opposes the supply voltage. This obeys Lenz's law. The induced, or back emf, creates an induced current, which opposes the supply current.

Example of Lenz's law.



Simple circuit with a coil having a low resistance, a typical lamp whose resistance increases as it gets warm, and a switch to control the current flow. The voltage is provided by a battery.

What do you think will happen when you close the switch?

From your work on resistors in parallel, you may think that the lamp will glow all the time the switch is closed. Wrong!

From your work on magnetism you learnt about inductance, we just need to revisit that to help you understand the effect an a.c. supply has on a coil.

When you close the switch the lamp will glow brightly because current will always take the easiest path, i.e. one that has less resistance to current flow.
Current is also trying to flow through the inductor, but as you remember, when a current flows through a coil, a magnetic field is created around the coil. Whenever a conductor is cut by lines of flux such as the ones created by the magnetic field, a voltage is induced which causes a current to flow. This current according to Lenz's law, will always flow in a direction to oppose the cause producing it. In other words it is reacting against being created, it is resisting the change. This causes a delay in the current flow through the coil. However, after a little time the field around the coil is established and current will flow normally through it. In fact, in its steady state it has a lower resistance than the path with the lamp.

In effect, on closing the switch the lamp is initially bright but then starts to go dim as the current in the inductor path establishes itself.

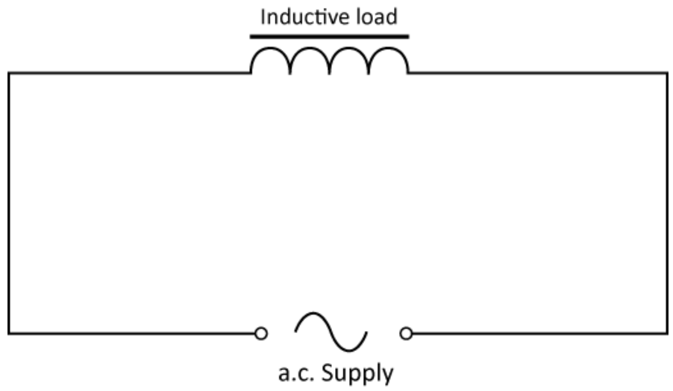
What happens when you open the switch?

This is the interesting thing about inductors; they don't want to commit suicide, they want to keep going. Remember, when current changes in a magnetic field, that is when it is growing or collapsing, an induced current will flow to try and oppose the change causing it. In the case of having the supply removed, the energy stored within the field will briefly cause a circulating current to flow which will make the lamp once again shine brightly before the energy is spent and the lamp goes out.

The level of inductance depends on the number of turns on the coil and the rate of change of flux (how quickly the magnetic field changes).

Rather than being bogged down in formulae (this will come later), the important thing to note is that the induced emf opposes the supply. This seems very strange as a very real second voltage is produced.

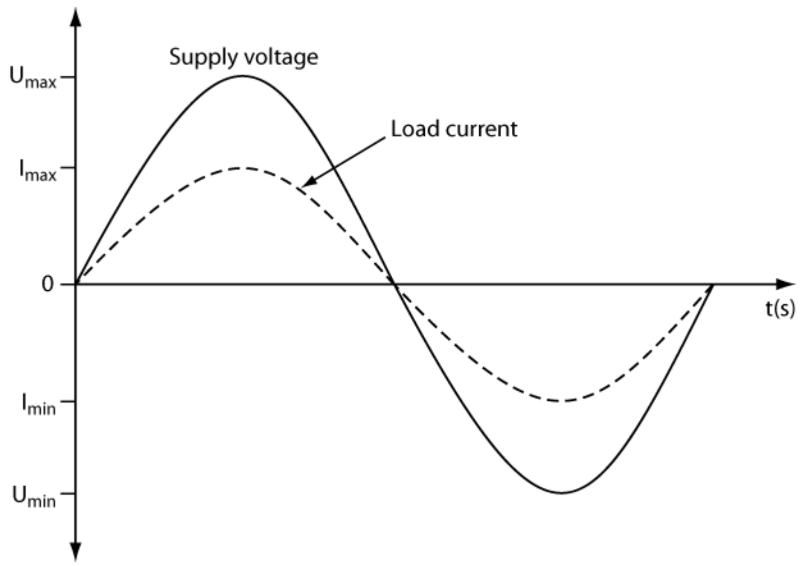
Now this inductive effect is compounded in an a.c. circuit.



Here we have our a.c. supply connected to an inductor. We will assume, for the moment that the inductor only has inductance and no resistance. This is impossible in real life; however it is useful to assume that we can have a pure inductor for clarity's sake.

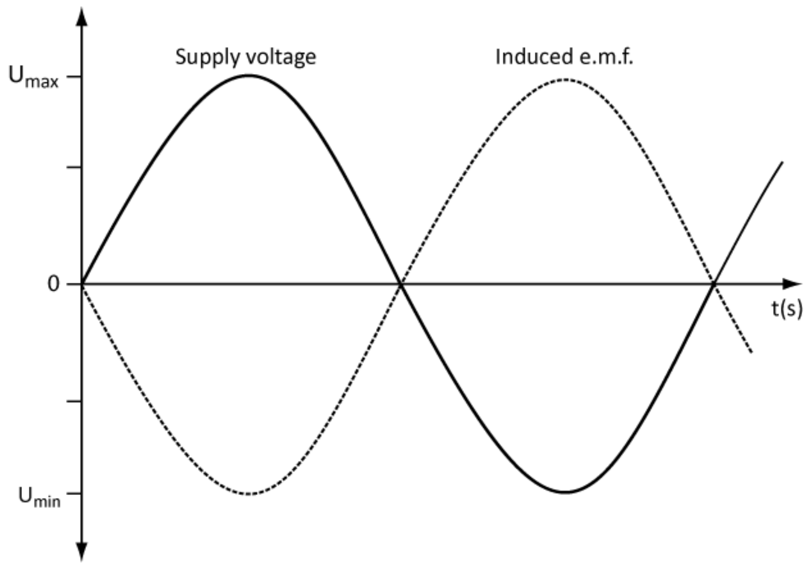
The supply current causes a magnetic field to be produced around the coil of the inductor. This coil then induces an emf back into the coil, which we call a ***back emf***. The back emf produces a current that opposes our supply current. This I have already stated.

The induced current however has a time delay in its effect on the supply current and we get what is called '***phase shift***'. The time delay is caused by the time taken for the magnetic field to build up. Effectively the supply current is no longer '***in-phase***' with the supply voltage, but has been shifted back 90°.



The above diagram shows the supply voltage and the supply current, if there is no inductive effect. You have seen this with a resistor in the circuit.

We will now assume that we have connected our supply to a pure inductor (one that has no resistance or capacitance). In this next diagram the supply voltage and the induced emf are exactly opposite to each other.

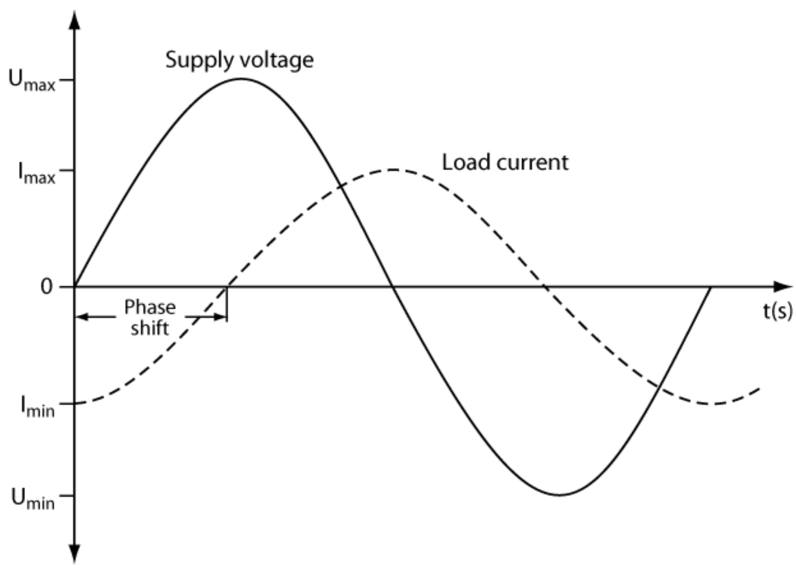


When the supply voltage is at a maximum, then the induced emf is at a minimum. Conversely, when the supply voltage is at a minimum then the induced emf is at a maximum.

The magnetic field in the inductor however takes some time to be created and the current, which in the purely resistive circuit is in-phase with the supply, gets delayed. This means that the current in the inductive circuit is '***out of phase***' with the supply voltage.

The inductor acts to delay the current and ***opposes*** any ***change in the current***. This is a very important principle and you must remember it.

Where the resistor opposes current flow, the ***inductor opposes current change***.



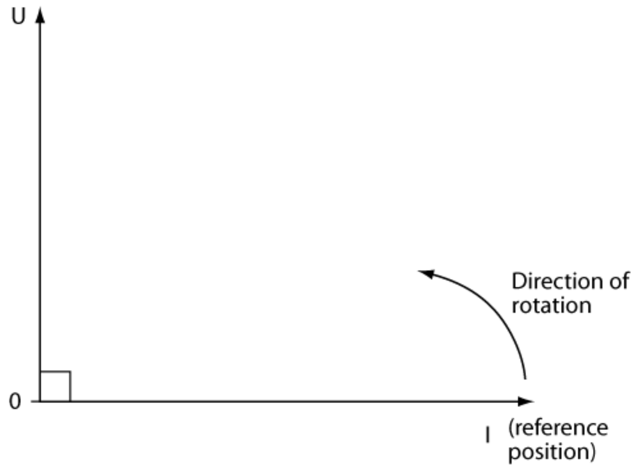
Here, you can see that the current waveform is '***lagging***' the voltage waveform by 90°, or a quarter of a cycle.

The maximum change on the supply voltage occurs when it passes through zero. This means that the current in an inductive circuit will be at a maximum at this point. Remember that it opposes change!

This also means that when there is no change, such as at the maximum or minimum value then there is zero current.

This effect can be drawn, as for the resistor, as a phasor diagram. Remember that the phasor diagram is a representation of the current and voltage in this particular instance.

Try to get into the habit of drawing the lines to scale. It will help you in the end with determining the combined values of other quantities.



Here you can see that the current is drawn along the horizontal axis, whilst the voltage is drawn rising vertically up from the current reference line.

The thing to remember is that a ***resistor opposes current flow*** and an ***inductor opposes current change***. The current flowing through a resistor is 'in-phase' with the supply voltage whilst the current through an inductor is 'lagging' the voltage by 90°, or a quarter of a cycle.

You are all aware of Ohm's law $U = IR$ and, the circuit above would not hold any problems for you, with an inductor we have to take into account other factors, and in particular the changes that occur with frequency.

Inductive reactance

Inductive reactance is a measure of the effect of the inductor on the a.c. circuit and we determine this using:

$$X_L = 2\pi f L$$

Where X_L = Inductive reactance (Ω)

f = Frequency (Hz)

L = Inductance (H)

We'll try a couple of examples to show you how this works.

1). A pure inductor has an inductance of 0.5 H. Determine the inductive reactance if the frequency is first 50 Hz and then 100 Hz.

i). $X_L = 2\pi f L$

$X_L = 2 \times \pi \times 50 \times 0.5 = \underline{\underline{157.1\Omega}}$

ii). $X_L = 2\pi f L$

$X_L = 2 \times \pi \times 100 \times 0.5 = \underline{\underline{314.2\Omega}}$

In this example notice, that as the frequency doubles so does the reactance. You should also notice that the reactance is still measured in ohms.

2). An inductor has a 60 Hz supply and has an inductive reactance of 25 Ω. What will be the inductance of the inductor?

$X_L = 2\pi f L$ transpose

$$L = \frac{X_L}{2\pi f}$$

$$L = \frac{25}{2 \times \pi \times 60}$$

$$L = \frac{25}{377} = \underline{\underline{0.066\text{H}}} = \underline{\underline{66\text{mH}}}$$

In this example, we have to transpose the formula to get our value of inductance.

We'll move on and look at how we can start to use our values of reactance.

The value of $\boldsymbol{X_L}$ can be used in a variation on Ohm's law.

$$U = IX_L$$

In this instance, $\boldsymbol{X_L}$ has replaced the resistance value. The working out of this type of problem is the same as it would be for a '***normal***' Ohm's law question.

We'll try an example.

1). A 220 V, 60 Hz a.c. supply is connected to a pure inductor. The inductor has an inductance of 0.5 H. Calculate the inductive reactance and the current flowing through it.

$$X_L = 2\pi fL$$
$$X_L = 2 \times \pi \times 60 \times 0.5 = \underline{\underline{188.5\Omega}}$$
$$U = IX_L \quad \text{transpose}$$
$$I = \frac{U}{X_L}$$
$$I = \frac{220}{188.5} = \underline{\underline{1.17\text{A}}}$$

In this example, we have first worked out the reactance. Remember that this is a measure of the opposition to current change in a circuit. After we know the reactance then we can determine the current flowing.

The next thought that you should have is that when frequency changes then there will be a corresponding change in the current. This is quite true. Look at the last example with a reduction in frequency to 50 Hz.

$$X_L = 2\pi fL$$
$$X_L = 2 \times \pi \times 50 \times 0.5 = \underline{\underline{157.1\Omega}}$$
$$U = IX_L \quad \text{transpose}$$
$$I = \frac{U}{X_L}$$
$$I = \frac{220}{157.1} = \underline{\underline{1.4\text{A}}}$$

Some quick maths tells us that for this reduction in frequency there has been nearly a 20% increase in current. This is why of course that supply generators are only allowed to vary the frequency of the supply by 1% or 0.5 Hz either side of 50 Hz.

2). A pure inductor of inductance 0.5 H is connected across a 230 V supply. Calculate the current flowing at a frequency of 50 Hz and at 25 kHz.

Current at a frequency of 50 Hz.

$$X_L = 2\pi fL$$

$$X_L = 2\times\pi\times 50\times 0.5 = \underline{\underline{157.1\Omega}}$$

$$U = IX_L \quad \text{transpose}$$

$$I = \frac{U}{X_L}$$

$$I = \frac{230}{157.1} = \underline{\underline{1.46\text{A}}}$$

Current at a frequency of 25 kHz.

$$X_L = 2\pi fL$$

$$X_L = 2\times\pi\times 25\times 10^3\times 0.5 = \underline{\underline{78540\Omega}}$$

$$U = IX_L \quad \text{transpose}$$

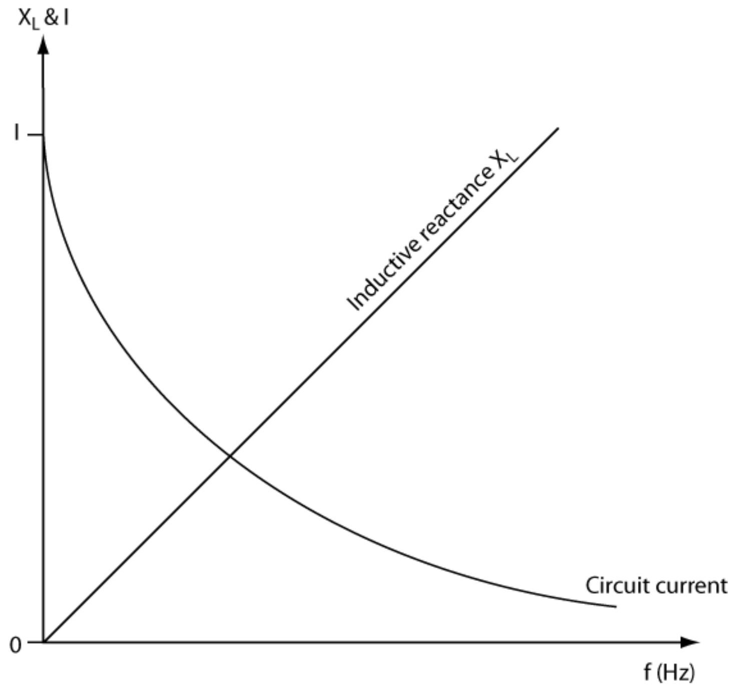
$$I = \frac{U}{X_L}$$

$$I = \frac{230}{78540} = \underline{\underline{2.93\text{mA}}}$$

The calculation at the top shows that when the frequency is at 50 Hz, the current is 1.46 A, whilst the calculation at the bottom shows that when the frequency rises to 25 kHz the current falls to less than 3 mA.

An increase in frequency leads to a reduction in the current. This effect has some very real practical benefits, particularly in the new, low power, mini-fluorescent all-in-one lamps, which run at a frequency of approximately 30 kHz. Notice also that this allows cable sizes to be dramatically reduced for the same output.

Have a look at the graph below.



The vertical axis shows reactance and current, whilst the horizontal axis shows the frequency.

As the frequency increases, then the reactance increases in direct proportion. We can state this because we are getting a straight line. We can say that the inductive reactance is directly proportional to the frequency, $f \propto X_L$; as the frequency increases the current falls, but not in a straight line.

If the frequency were to fall to zero, which would effectively give us d.c., then the current would be infinite. This is nonsense. What would actually happen is that the supply would simply be looking at the resistance of the conductors and not the reactance, because we can never get a pure inductor!

If the frequency were to increase to infinity, then the current would fall to zero. This is more likely to occur. We can already see that at high frequencies such as 25 kHz, the supply current would fall to a very low figure.

Exercise 3.

1) A 0.1H inductor is connected across a 200 V, 60 Hz a.c. supply. What is the current drawn from the supply?

2) A 5 mH inductor is connected across a 12 V, 5 kHz a.c. supply. What is its current?

3) An inductor when it is connected across a 24 V 50 kHz a.c. supply draws a current of 6 mA. What is the value of the inductor?

4) In a circuit that consists of a pure inductor, what happens when the frequency increases?

5) A pure inductor has an inductance of 15 mH. What will be the current flowing in the circuit if the supply has an initial frequency of 50 Hz and then one of 2.5 kHz? Assume that the supply voltage is 48 V?

6) A pure inductor operates at a frequency of 50 Hz. If it has an inductive reactance of 250 Ω, what will be its inductance?

7) A coil having an inductance of 0.2 H is connected across a 230 V 50 Hz supply. What will be the current flowing in the circuit, if we assume that there is no resistance?

4: Effects of a capacitor in an a.c. circuit

In this session the student will:

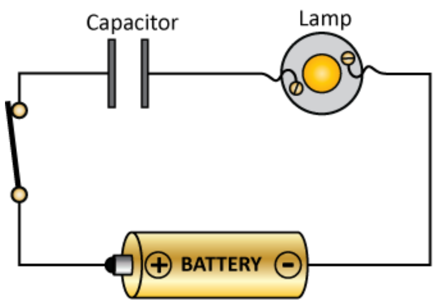
- Describe the effect a capacitor has on an a.c. circuit.

We have now looked at a pure resistor and a pure inductor. We know that it is almost impossible to have a pure anything. However, to give a clearer understanding of what is happening, we make certain assumptions; purity is one of these assumptions. We are now going to look at what happens when a pure capacitor is connected in an a.c. circuit.

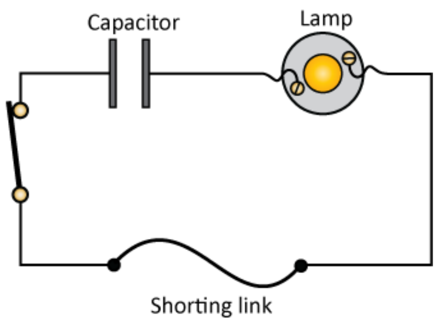
In the last session we saw that when an inductor is connected into a circuit then it causes the current in the circuit to be delayed – it lags the supply voltage. This is a natural effect of this component. Things are exactly the same with a capacitor. A capacitor has the greatest current flowing when the supply is first turned on.

From the previous module on capacitors, you learnt that when a capacitor is connected to a d.c. supply there is an initial current flow until the capacitor becomes charged, this then prevents any further current flow.

Consider the d.c. example shown below.

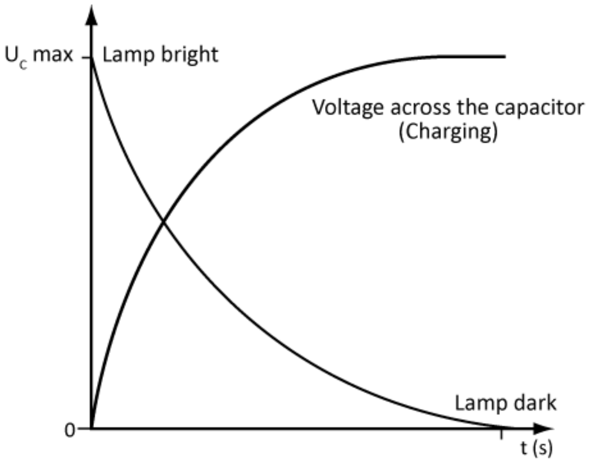


On closing the switch there will be an initial current flow to light the lamp, then after a period of time the lamp will dim and go out. The capacitor is now charged.

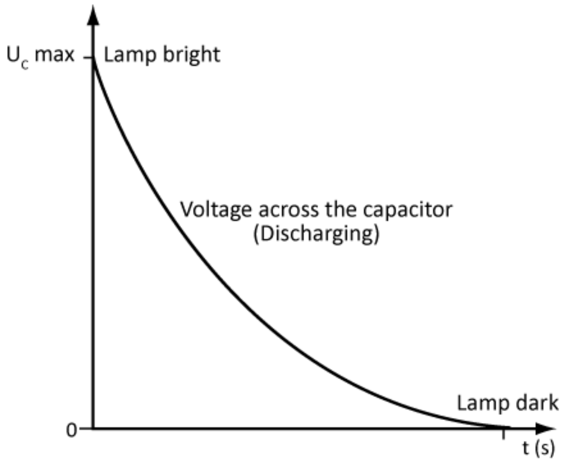


If the battery is replaced with a shorting link, there will be an initial current flow to light the lamp, then after a period of time the lamp will dim and go out. The capacitor is now discharged.

We can show this effect using graphs.

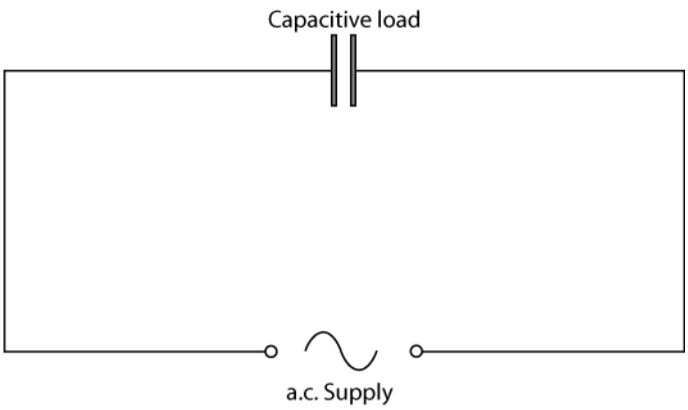


When the battery is connected, the lamp is lit until the capacitor becomes charged, then it appears to be an open-circuit as no more current will flow. The lamp is now out.



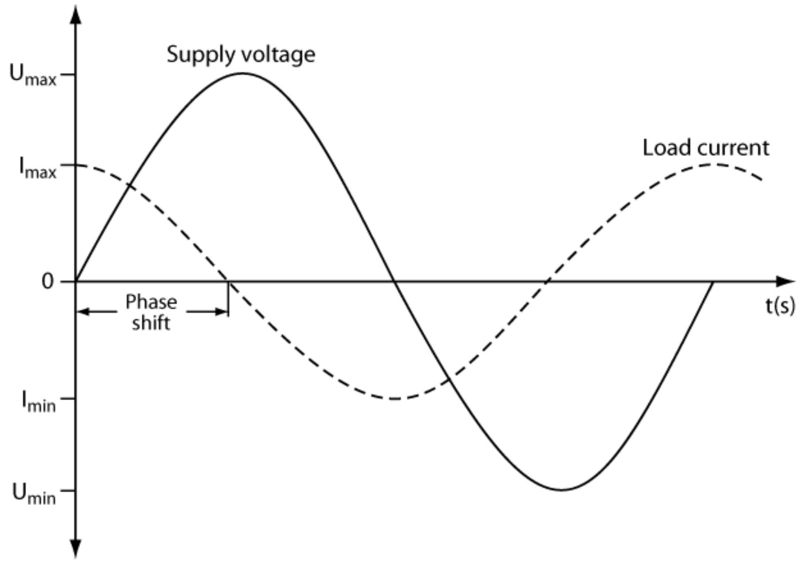
When the battery is dis-connected and replaced with a shorting link, there will be an initial flow of current (in the opposite direction), and the lamp will be lit. Eventually the voltage stored in the capacitor will become exhausted and the lamp will go dim then dark.

If we now take this idea and apply it to an a.c. circuit, what will happen is that the capacitor is charging and discharging every time the alternating supply moves from its positive half cycle to its negative half cycle. Have a look below.



In this circuit the capacitor is first charged by the positive half cycle and then discharges when the negative half-cycle comes along. As you might expect this charging and discharging of the capacitor has an effect on the supply current. Remember, with a d.c. circuit there was current flow first, the voltage across the capacitor grew to limit it.

We'll look at what happens on the first half cycle of the supply and you should get some idea of what effect there is on the a.c. current. Look below.



You can see the a.c. waveform produced by the voltage supplied to the circuit. The current is in front of the voltage.

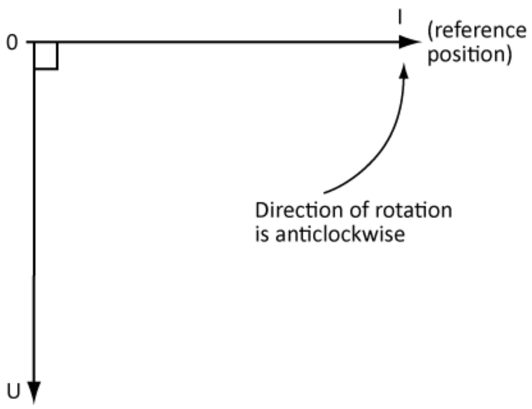
We now have to consider what happens to a capacitor when it is being charged up.

When the supply is first switched on, at that very instant, there are zero volts applied to the capacitor. At that same point however, the charging current is at a maximum.

As the capacitor charges up, the voltage across it rises and the current begins to fall to the point where the voltage across the capacitor reaches a maximum and the current reaches 0 A (the capacitor is fully charged).

This effect causes the current in the capacitor to '***lead***' the voltage by 90° or a quarter of a cycle.

This relationship can be drawn using a phasor diagram as we have done for the resistor and the inductor; this time however the current is shown leading the voltage.



Notice that the phasor diagram for the capacitor is drawn exactly opposite to the phasor diagram of the inductor.

In an a.c. circuit the capacitance does tend to counteract the effect of the inductor in the circuit, and this effect is made use of in power factor correction, which we will look at later on.

For the capacitor:

$$X_c = \frac{1}{2\pi fC}$$

Where $X_c =$ Capacitive reactance (Ω)

$f =$ Frequency (Hz)

$C =$ Capacitance (F)

We are already aware that a capacitor connected into an a.c. circuit affects the circuit in a similar way to the inductor. The capacitor, as with the inductor, opposes any change in the current in the supply.

Again, we'll consider the pure capacitor. As you would expect there is a formula attached to this. The formula can be proved, however at present it is unnecessary to look at where it comes from. You should just accept that it is true.

As with the inductors, we'll look at how this works out with a few examples.

1). An 8.7 µF capacitor is connected across an a.c. supply. What is the capacitive reactance if the frequency is 60 Hz?

$$X_c = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{2\times\pi\times 60\times 8.7\times 10^{-6}}$$

$$X_c = \frac{1}{3.28\times 10^{-3}}$$

$$X_c = \underline{\underline{304.9\Omega}}$$

Here the reactance is again measured in ohms, as it was with inductive reactance.

The part of this that can be a little confusing is the inverting of the bottom line. Try another one.

2). A 0.1 µF capacitor is connected across an a.c. supply. The frequency is set at 50 Hz and then at 15 kHz. What is the reactance at the two frequencies?

i). $$X_c = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{2\times\pi\times50\times0.1\times10^{-6}}$$

$$X_c = \frac{1}{3.142\times10^{-5}}$$

$$X_c = \underline{\underline{31831\Omega}}$$

ii). $$X_c = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{2\times\pi\times15\times10^{3}\times0.1\times10^{-6}}$$

$$X_c = \frac{1}{9.42\times10^{-3}}$$

$$X_c = \underline{\underline{106.1\Omega}}$$

Again, you should notice that frequency plays the largest part in the effect on the reactance in the circuit.

3). The capacitive reactance of a circuit is 250 Ω. If the frequency of the supply is 50 Hz, what will be the capacitance value of the capacitor?

$$X_c = \frac{1}{2\pi fC} \quad \text{transpose}$$

$$C = \frac{1}{2\pi fX_c}$$

$$C = \frac{1}{2\times\pi\times50\times250}$$

$$C = \frac{1}{78540}$$

$$C = \underline{\underline{0.0000127\text{F}}} = \underline{\underline{12.7\mu\text{F}}}$$

The transposing is not too difficult. You should notice that I have just interchanged the reactance (***X_C***) with the capacitance value.

As with inductive reactance there is a similarity to Ohm's law, when we are dealing with how the reactance relates to the current and voltage.

$$U = IX_c$$

Again, this is straightforward and should be used as you would use Ohm's law (i.e. $U = IR$).

4). A pure capacitor of capacitance 2.5 µF is connected across a 220 V supply. Calculate the current flowing at a frequency of 50 Hz and at 25 kHz.

At a frequency of 50 Hz

$$X_c = \frac{1}{2\pi fC} \quad \text{transpose}$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 2.5 \times 10^{-6}}$$

$$X_c = \frac{1}{7.85 \times 10^{-4}}$$

$$X_c = \underline{\underline{1273\Omega}}$$

$$U = IX_c \quad \text{transpose}$$

$$I = \frac{U}{X_c}$$

$$I = \frac{220}{1273} = \underline{\underline{0.173\text{A}}} = \underline{\underline{173\text{mA}}}$$

At a frequency of 25 kHz

$$X_c = \frac{1}{2\pi fC} \quad \text{transpose}$$

$$X_c = \frac{1}{2 \times \pi \times 25 \times 10^{-3} \times 2.5 \times 10^{-6}}$$

$$X_c = \frac{1}{0.393}$$

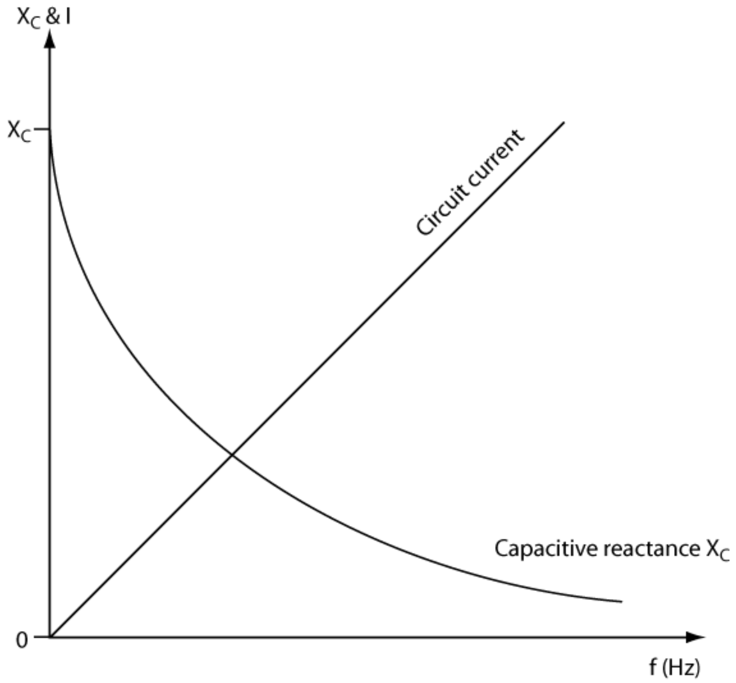
$$X_c = \underline{\underline{2.55\Omega}}$$

$$U = IX_c \quad \text{transpose}$$

$$I = \frac{U}{X_c}$$

$$I = \frac{220}{2.55} = \underline{\underline{86.4\text{A}}}$$

At the top, the frequency of 50 Hz creates a relatively small current, whereas at 25 kHz, at the bottom, the current has risen dramatically. A graph expresses this information, as with the inductive reactance.



The vertical axis shows reactance and current, whilst the horizontal axis shows the frequency. As the frequency increases, then the current increases in direct proportion. We can say that the circuit current is directly proportional to the frequency, $I \propto f$.

As the frequency increases the reactance falls, however this is not a linear relationship and there is not a straight line. If the frequency were to fall to zero, which is in effect d.c., then the reactance would be infinite, and current would be blocked. This is exactly what happens. In fact, in electronic circuits capacitors are used to block direct current. You saw this earlier with the simple lamp circuit.

If the frequency were to increase to infinity, then the reactance would fall to zero and the current would correspondingly increase.

This is exactly the opposite of what occurred with the inductive reactance.

Exercise 4.

1) A pure capacitor is connected first to a 60 Hz supply and then to a 30 kHz supply. If it has a capacitance value of 4 µF and a supply voltage of 25 V, what will be the current flowing in the circuit?

2) What will happen to the circuit in Q.1 if the frequency fell to 50 Hz?

3) A 10 µF capacitor is connected into a circuit which has 10 mA_{rms} flowing. If the frequency of the supply is 1 kHz, determine the supply voltage and the voltage rating of the capacitor to prevent its dielectric breaking down.

4) Fill in the table below for pure capacitance.

Frequency	Voltage	Capacitance	Current	Reactance
1 kHz	48 V	200 nF		
50 Hz	110 V		1.2 A	
	230 V	8 µF	0.85 A	
	50 V	1 mF		100 Ω

5: Effects of impedance in an a.c. circuit 1

In this session the student will:

- Gain an understanding of what impedance is.
- Be able to describe the combined effect of resistance and inductive and capacitive reactance in an a.c. circuit.

We've already said that in the real world there is no such thing as a pure anything. This is certainly true for the inductor, although the capacitor is as near pure as we are likely to get.

The inductor is made up of a large number of turns of wire. This means that there must be some resistance due to the length of wire used. When things get warmer an inductor's resistance also increases.

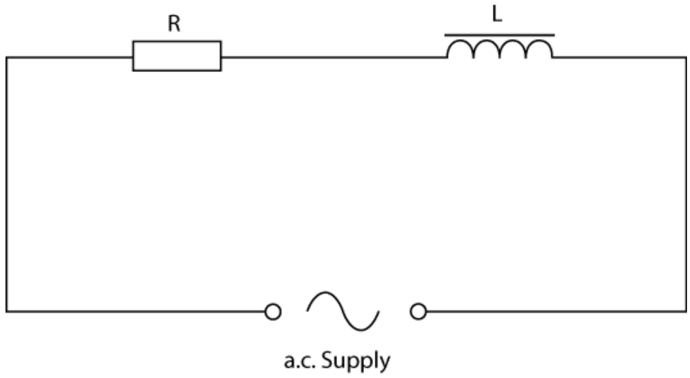
When we look at real circuits then, we are often faced with inductors connected with resistors and capacitors. We'll look at some of these variations.

When you were looking at resistors connected in series in d.c. circuits, you will remember that the current remained constant throughout the circuit. When you look at series circuits supplied from an a.c. source, you should still remember those same basic rules about series circuits – the current is constant!

In a series circuit, the ***current is constant***. This is true for a.c. and d.c. There are some differences in how the current is determined in the a.c. circuit when compared to a d.c. circuit however.

Inductors and resistors connected in series

Have a look at the diagram below. Here we have an inductor and a resistor connected in series.



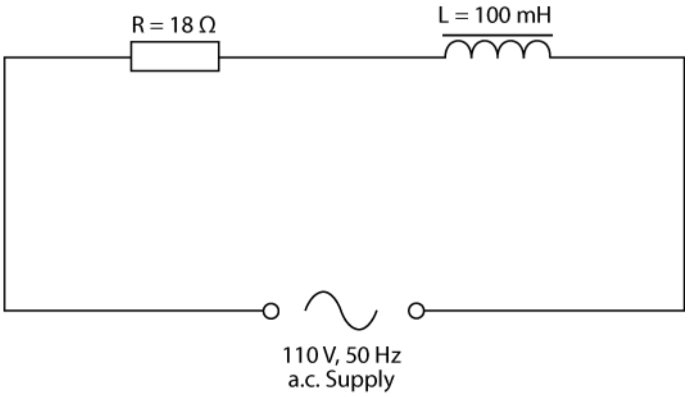
This would be the normal state of affairs. An inductor will always have some resistance. When we try to work out what is happening we have to consider two effects of reactance and resistance. This is why in a diagram it is easier to separate out the individual parts and treat them as two distinct parts of the circuit, even though they are part of the same piece of equipment.

Earlier in this outcome, we looked at the effect on the supply of a pure inductor, and we saw that the current lagged the voltage by 90°. This was because the inductor opposed current change. We also saw that the effect of a pure resistor on the supply as it opposed the current flow, and you had phasor diagrams introduced to you. It is now that they come into their own as we see the combined effect of the two effects together.

We'll work through some examples and see how certain rules and techniques apply. The first example is rather spread out. This is done to try to show in detail the particular steps necessary. The other examples will follow later. Take your time over this section, it is important!

1). An 18 Ω resistor is connected in series with a 100 mH inductor, to a 110 V, 50 Hz, a.c. supply. Determine the current and the total effect of the reactance and the resistance.

It is always worthwhile doing a drawing.



We know that an inductor acts differently to a resistor, but we can deal with each individually, and so avoid confusion. In this instance, we know what the resistance is, but we need to find out what the reactance is. (Remember that reactance is the opposition to current change in an a.c. circuit).

We **cannot** just add the resistance and reactance together as we know that they are 90° out of phase with each other. There needs to be some other mechanism used.

Have a look at the first part of the working out below.

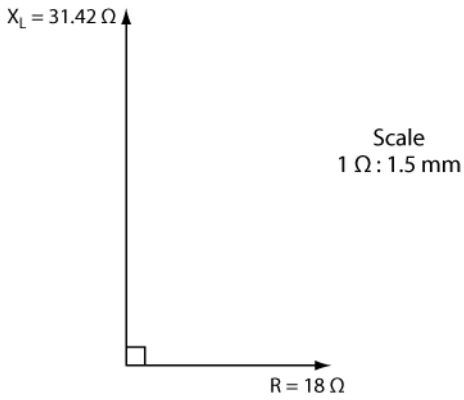
$$X_L = 2\pi fL$$
$$X_L = 2 \times \pi \times 50 \times 100 \times 10^{-3}$$
$$X_L = \underline{\underline{31.42\Omega}}$$

This working out shows the reactance of the inductor.

Now as we have already seen, Pythagoras said that ***the square on the hypotenuse is equal to the sum of the squares on the other two sides***. This means that we have two ways in which we can determine the impedance.

Method 1 – use of a scale diagram

Below we have two lines drawn. The horizontal line drawn to scale expresses the resistance (in phase with the supply) and should reflect a resistance of 18 Ω. I suggest a line of 27 mm.



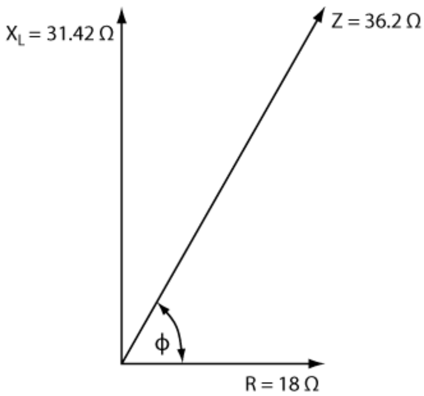
The vertical line (pointing upwards) expresses the reactance value of 31.42 Ω (90° out of phase with the supply), and again should be drawn to scale. My line is 47.13 mm long.

You should notice that the two lines are drawn 90° apart. Effectively we have a right-angled triangle and we can use the techniques gained from Pythagoras to determine the overall effect.

This overall effect of combining the reactance and resistance of a circuit is called the '***impedance***' of the circuit. Impedance is an a.c. effect and it doesn't occur in d.c. circuits. The label attached to impedance is '***Z***'.

All we have to do now is complete the triangle.

If you have drawn the diagram to scale then the hypotenuse is an accurate measure of the impedance of the circuit. The impedance, as you will remember, is the overall effect of the resistance and the reactance in the circuit.



The total impedance in our example should come to approximately 54mm as measured by your ruler. This length reflects a value of 36Ω.

Method 2 – by calculation

The second way of operating is to calculate it. We know about the right-angled triangle and so we know that:

$$\text{hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$$

In the electrical industry, we don't use terms such as the hypotenuse etc. For our circuits we have the following:

$$R = \text{Resistance} = \text{adjacent side}$$
$$X_L = \text{Reactance} = \text{opposite side}$$
$$Z = \text{Impedance} = \text{hypotenuse}$$

So we have:

$$Z^2 = R^2 + X^2$$
$$Z = \sqrt{R^2 + X^2}$$

Remember that we are still dealing with our first example, and going back to it.

Circuit impedance.

$$X_L = 2\pi fL$$
$$X_L = 2 \times \pi \times 50 \times 100 \times 10^{-3}$$
$$X_L = \underline{\underline{31.42\Omega}}$$
$$Z = \sqrt{R^2 + X_L^2}$$
$$Z = \sqrt{18^2 + 31.42^2}$$
$$Z = \sqrt{1311.2164} = \underline{\underline{36.211\Omega}}$$

The calculation gives us a precise answer. The diagram drawn to scale is only as accurate as you are neat and tidy.

In this problem however, we now know the overall effect of the resistance and the reactance on the current. This means that we can work out the current.

Once the total impedance is found then the current can be found using the variant of Ohm's law.

$$I = \frac{U}{Z}$$

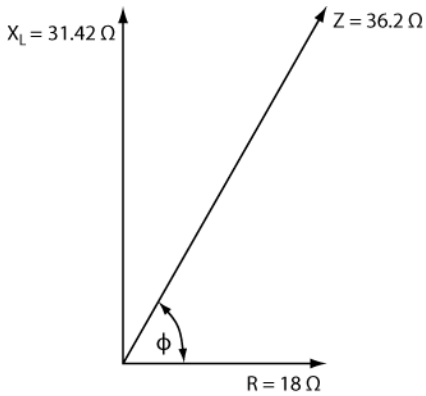
$$I = \frac{110}{36.211} = \underline{\underline{3.04\text{A}}}$$

Notice the process.

- **Inductive reactance**
- **Impedance**
- **Current.**

The current can only be worked out if the impedance has been found in this example. The total effect of a resistor and an inductor, combined in an a.c. circuit, is greater than each one on its own would produce. Take care – we cannot have two separate currents in a series circuit!

We don't need to stop here however. There is an important lesson to be learnt. Have a look at the completed diagram again.



You can see that an angle is created between the impedance line and the resistance line. This angle tells us the amount of '***shift***' that has occurred between the norm (the resistance) and the inductor that has caused the shift.

We'll work out the angle. You should remember that when we are looking at a right-angled triangle we can find the angle if we know two of the side lengths.

Now in our examples we know all three lengths and so we have a whole range of sides that we can choose from. We'll look at our options.

$$\cos\phi = \frac{A}{H} = \frac{18}{36.211} = 0.4971 \qquad \sin\phi = \frac{O}{H} = \frac{31.42}{36.211} = 0.8677 \qquad \tan\phi = \frac{O}{A} = \frac{31.42}{18} = 1.745$$

$$\phi = \cos^{-1} 0.4971 = \underline{\underline{60.193^\circ}} \qquad \phi = \sin^{-1} 0.8677 = \underline{\underline{60.192^\circ}} \qquad \phi = \tan^{-1} 1.745 = \underline{\underline{60.192^\circ}}$$

You can see that it doesn't matter what sides you choose. You must just make sure that the sides are properly labelled. Don't get bogged down in using all three methods. Pick one and stick with it. The cosine variant is normally used, but it doesn't really matter. Do not be too quick to round up or down early into the calculation, it leads to the final answer not being as accurate as it could be.

The opposite side is opposite the angle that you are looking for. The adjacent side is adjacent or next to the angle that you are looking for. The hypotenuse is always the longest side.

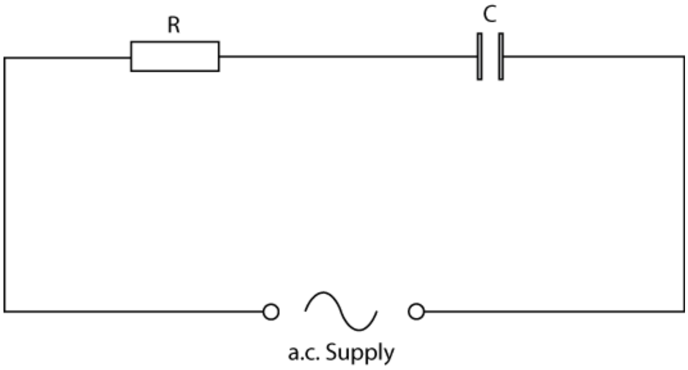
You should also remember that the cosine, sine or tangent is just a relationship between two sides. To get the actual angle, we have to use the inverse or second function button on the calculator combined with the sin/cos/tan button.

We call this angle, the ***phase angle***.

Try to get the answers that I get. The reason for the slight variation in the answers is the rounding up or down that the calculator makes. We'll look at this in more detail as we progress.

Capacitors and resistors connected in series

Have a quick look at the diagram below.

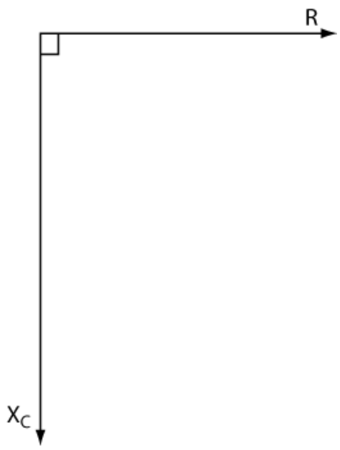


Here we have a resistor connected in series with a capacitor. Again we already know from the work that has been done with series circuits, that the one constant is the current that flows.

When we are considering capacitors and resistors in series, we operate in the same way as we did for inductors and resistors in series. We work out the capacitive reactance and combine it with the resistance value to find the impedance. We don't look for the overall resistance, but rather we look for the total impedance, ***Z***. This time however, we use $\boldsymbol{X_C}$, for capacitive reactance.

Resistance is determined in just the same way, using Ohm's law, and you have already seen how we can determine the capacitive reactance.

When we looked at the inductor/resistor network, we saw that the current '***lagged***' the voltage. We then drew a phasor diagram to describe how this appeared. With a capacitor/resistor network, the current '***leads***' the voltage, and again we can draw a phasor diagram to show this.



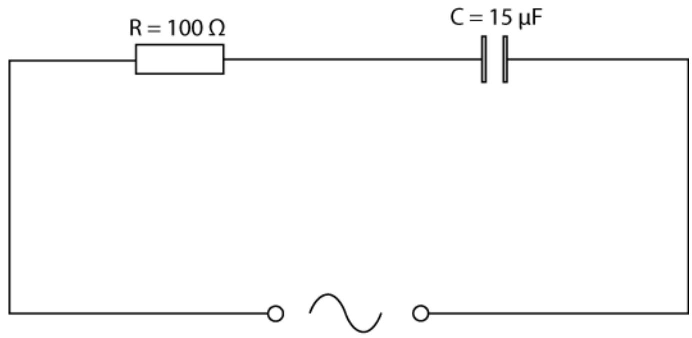
Here, you can see that the value produced by the capacitive reactance is drawn vertically downwards rather than upwards, as it was with our inductive reactance. The resistance remains the reference point along the horizontal.

The way of working out of this type of problem is virtually the same as that used when working out inductor/resistor circuits.

We'll go through a couple of examples.

2). A 100 Ω resistor and a 15 µF capacitor are connected in series across a 230 V, 50 Hz supply. Calculate the:

a) capacitive reactance
b) impedance
c) total supply current
d) phase angle in degrees.



The working out is shown below. The process to follow is the same as for inductors and resistors in series.

a) Calculate reactance
b) Calculate impedance or draw to scale and measure impedance
c) Calculate current
d) Determine the phase angle – either measured by drawing to scale or calculated.

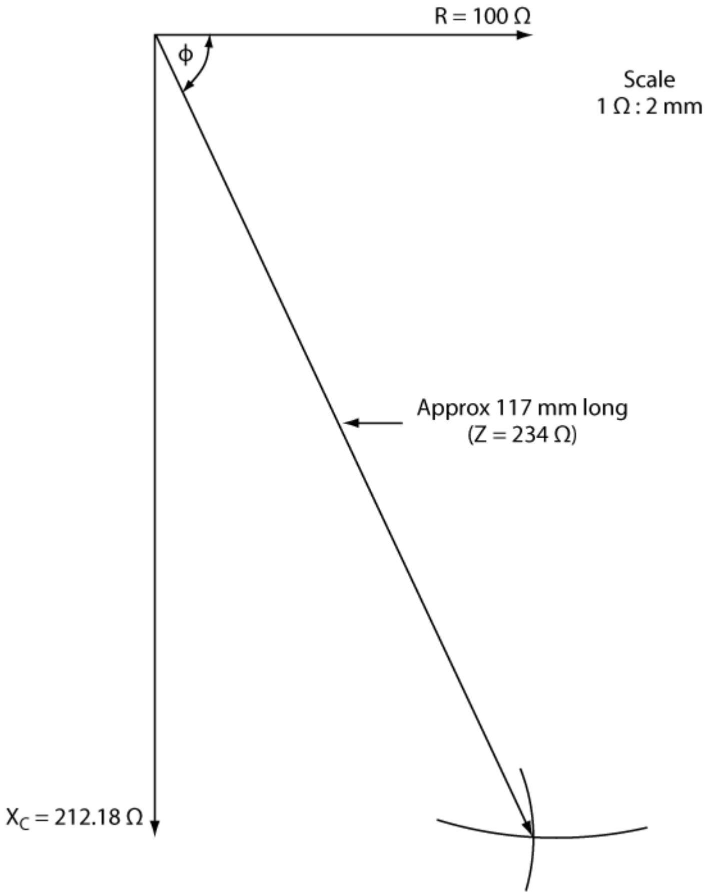
a). $X_C = \dfrac{1}{2\pi f C_C} = \dfrac{1}{2\times\pi\times50\times15\times10^{-6}} = \underline{\underline{212.18\Omega}}$

b). $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 212.18^2} = \sqrt{55020} = \underline{\underline{234.56\Omega}}$

c). $I = \dfrac{U}{Z} = \dfrac{230}{234.56} = \underline{\underline{0.98\text{A}}}$

d). $\cos\phi = \dfrac{R}{Z} = \dfrac{100}{234.56} = 0.426 \quad \textit{giving} \ \phi = \cos^{-1}0.426 = \underline{\underline{64.77^\circ}}$

Have a look at the phasor diagram of this problem.



This time, notice the direction of the diagram. The capacitive reactance points downwards.

Exercise 5.

1) A resistor of 24 Ω and an inductor of 50 mH are connected in series. The frequency is 60 Hz and the supply voltage is 110 V. Determine:

   i) Reactance
   ii) Impedance
   iii) Current flow
   iv) Phase angle.

2) A 40 Ω resistor and an inductor of 0.1 H are connected in series. The frequency is 50 Hz and the supply voltage is 220 V. Determine:

   i) Reactance
   ii) Impedance
   iii) Current flow
   iv) Phase angle.

3) A 25 μF capacitor and a 65 Ω resistor are connected in series. The frequency is 50 Hz and the supply voltage is 230 V. Determine:

i) Reactance
ii) Impedance
iii) Current flow
iv) Phase angle.

4) A 25 Ω resistor is connected in series with a 150 nF capacitor. The frequency is 120 kHz and the supply voltage is 12 V. Determine:

i) Reactance
ii) Impedance
iii) Current flow
iv) Phase angle.

5) A coil of wire has a resistance of 8 Ω and an inductance of 80 mH. It is connected to a 230 V 50 Hz supply. What happens if the frequency doubles?

6) Two coils are connected in series. The following values for each coil apply:

Coil 1: R=15 Ω;
L=0.2 H.

Coil 2: R=25 Ω;
L=40 mH.

They are connected across a 230 V 50 Hz supply. Determine:

i) Impedance
ii) Current
iii) Phase angle.

6: Effects of impedance in an a.c. circuit 2

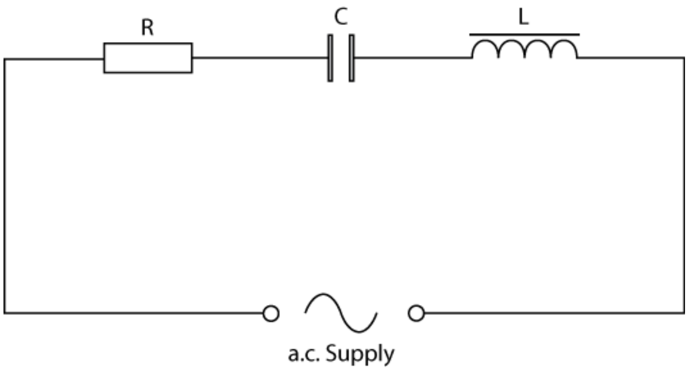
In this session the student will:

- Be able to describe the combined effect of resistance and inductive and capacitive reactance in an a.c. circuit.
- Gain an understanding of resonance.

In the last session we saw something of the combined effect of the resistor/inductor combination and the resistor/capacitor combination. In this session we are going to consider what happens when all three elements are combined. This is not as far-fetched as you might think. Consider a single-phase capacitor start induction run motor. The start winding of that particular motor is exactly what we are effectively considering in this session.

Capacitors, inductors and resistors connected in series

We've looked at resistors and inductors in series with each other. We have also looked at resistors and capacitors connected in series with each other. Now we are going to look at what happens when we connect resistors, inductors and capacitors together in series. Have a look at the diagram below.

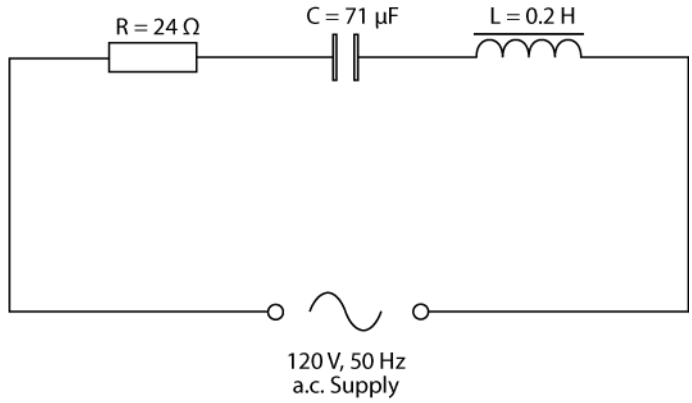


You can see that there is an inductor, resistor and capacitor. Remember that we are still looking at a series circuit and this means that the ***one constant feature is the current***. Whatever happens with everything else, never forget the constant current in a series circuit!

When we look at our first example, you should not believe that things have got much more complicated, they haven't! As long as you deal with each item in turn, then everything will drop out in the end.

You should also be aware that if the inductive reactance and the capacitive reactance are equal then they cancel each other out and can be effectively ignored, but more of this later. Consider the example below.

1). A 71 μF capacitor is connected in series with a 0.2 H inductor and a 24 Ω resistor. If the frequency is 50 Hz and the supply voltage is 120 V, what will be the total impedance, current and phase angle?



Remember to deal with each component in turn.

a) Calculate each reactance
b) Calculate impedance
c) Calculate current flow
d) Calculate phase angle.

a) $X_L = 2\pi f L_L = 2 \times \pi \times 50 \times 0.2 = \underline{\underline{62.83\Omega}}$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 71 \times 10^{-6}} = \underline{\underline{44.83\Omega}}$$

X_C and X_L are opposite

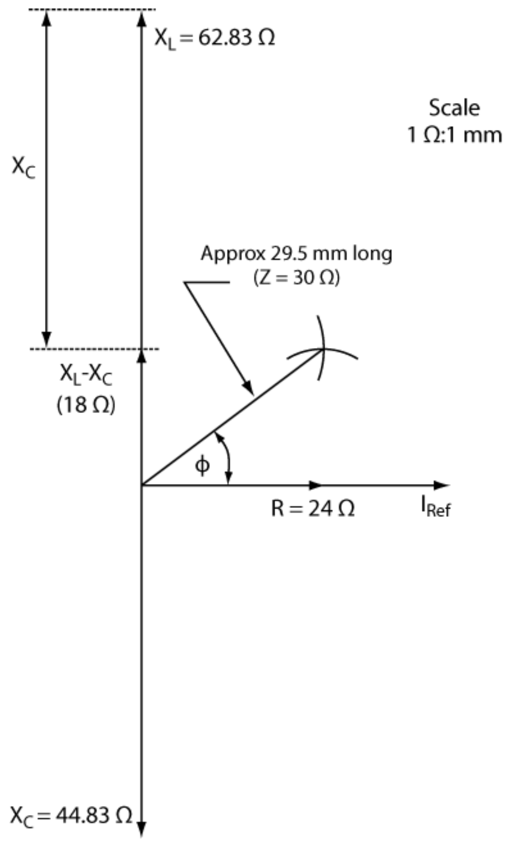
$\therefore$ Total $X = X_L - X_C = 62.83 - 44.83 = \underline{\underline{18\Omega}}$

b) $Z = \sqrt{R^2 + X_L^2} = \sqrt{24^2 + 18^2} = \sqrt{900} = \underline{\underline{30\Omega}}$

c) $I = \frac{U}{Z} = \frac{120}{30} = \underline{\underline{4A}}$

d) $\cos\phi = \frac{R}{Z} = \frac{24}{30} = \underline{\underline{0.8}}$ phase angle $\cos^{-1} 0.8 = \underline{\underline{36.87^\circ}}$

Have a look at the completed phasor diagram of the resistance/reactance below.



Note that the overall effect is to reduce the total reactance. The capacitive and inductive reactance values oppose, and partially cancel each other out.

There is a special case that occurs when the two reactances are the same value. When the two values are equal in value the circuit is said to be at resonance.

Resonance

Resonance is a strange thing, and occurs when the inductive reactance and the capacitive reactance are equal.

> Imagine that you are driving a car along the motorway. As you pick up speed your Corsa 'cruises' nice and quietly. At a certain speed, say 50mph, things start to rattle a little. As you move up through 60 mph your Corsa is now shaking quite a bit, and everyone is talking with a stammer. However, as you move beyond 70 mph (odd thing for a Corsa I hear you say) things start to quieten down a little, the shaking stops. The car doesn't go any faster however!

Think about resonance a little differently.

Why should the car begin to rattle and then stop once a certain speed has been reached?

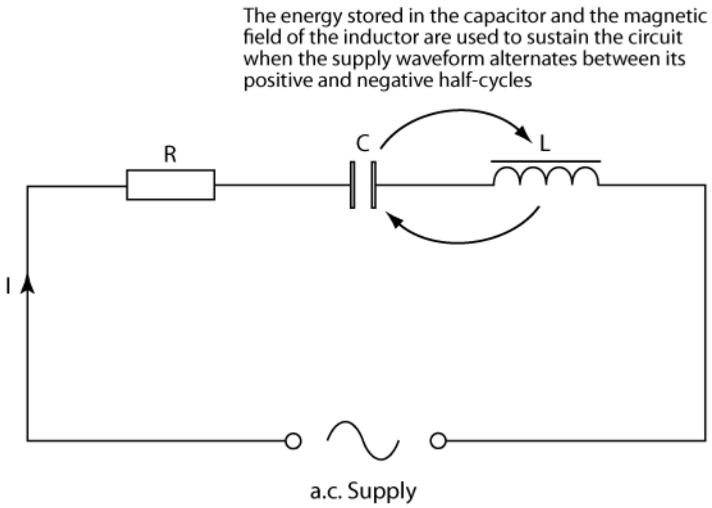
Everything has a natural frequency at which it begins to rattle. Usually everything rattles at a different frequency. However, occasionally everything rattles at the same frequency and things begin to drop off, they literally rattle loose. They are resonating!

> The Tacoma Narrows Bridge in the USA is a famous example of the effects of resonance. When a wind of a certain speed blew from a certain direction for a period of time the bridge begin to shake and eventually it started to concertina, almost like a snake. The bridge eventually collapsed. It was suffering from resonance.

In electrical terms, we have to consider resonance, and as with so many other areas of engineering it is frequency dependent.

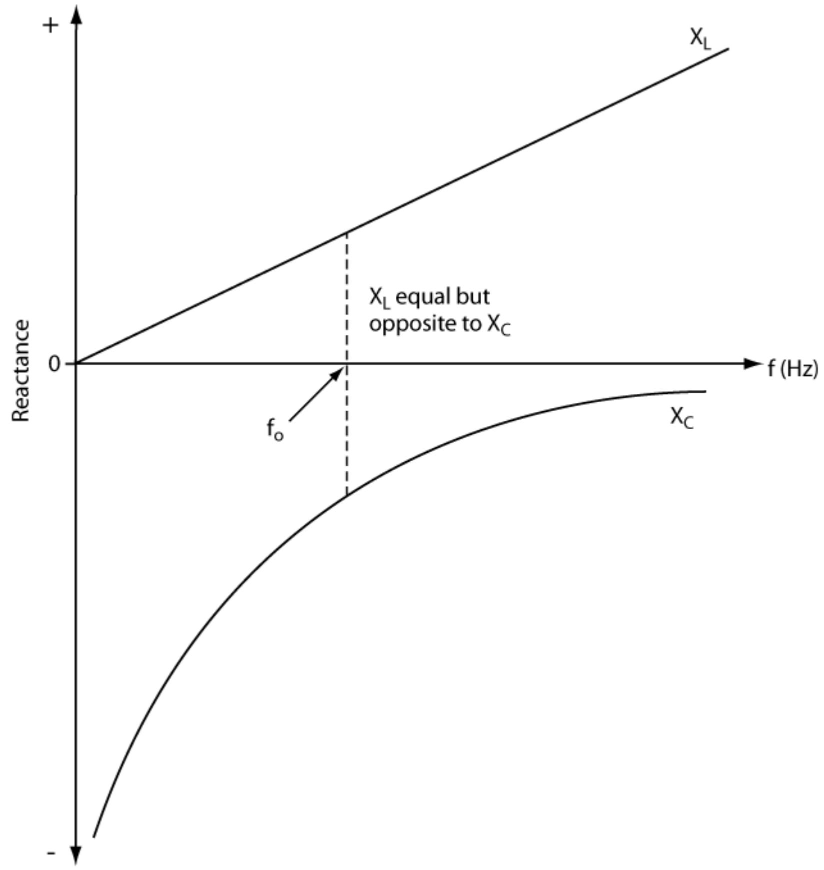
If we get back to considering our series circuits. Resonance occurs when the only energy required is that used to drive the resistor. The inductor and the capacitor are busy feeding one another, and simply charging and discharging into each other.

Let's take a slightly more detailed look at what happens.



We know that as the frequency increases then resistance remains constant. It is not frequency dependent. However, as frequency increases then inductive reactance increases whilst capacitive reactance decreases.

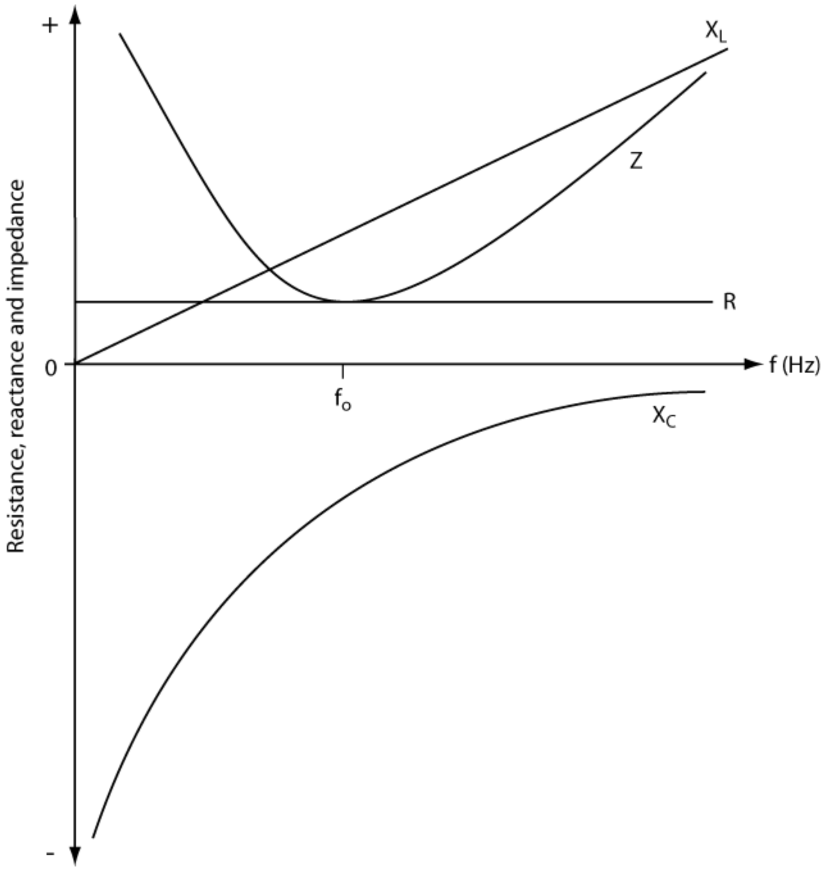
Now if we were to draw a graph and place inductive reactance and capacitive reactance on the same scale we would get something like the diagram over the page.



Now, the reason I have started the capacitive reactance below the line is that, although it is measured in ohms, as is inductive reactance, it is out of phase with inductive reactance by 90°.

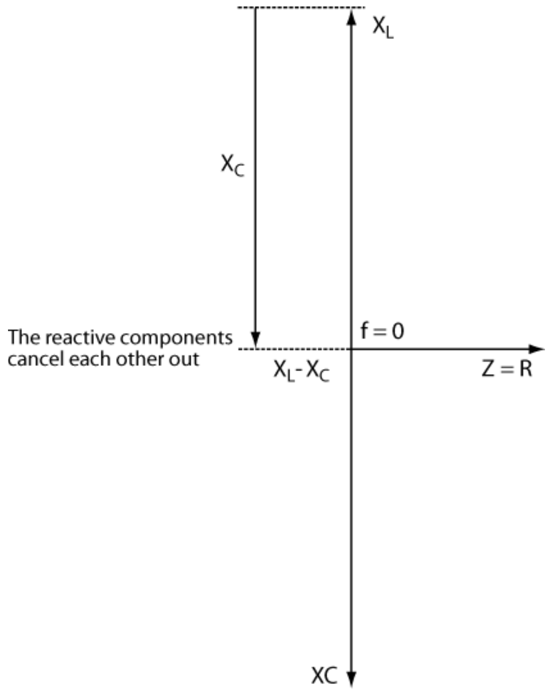
There does come a point where they cancel each other out, and this is where resonance occurs.

Below we can see all of the relevant parts including resistance and impedance.



When the reactance values become equal then they effectively cancel each other out and the only factor that affects current or voltage is the resistance of the circuit.

Have a look below.



You can see that the two reactance values, have cancelled each other out, and only the resistance is '***seen***'.

This can be represented mathematically.

At series resonance, as we have already stated inductive reactance and capacitive reactance are equal and opposite. In addition, the voltage dropped across the inductor is equal and opposite to the voltage dropped across the capacitor.

So when $X_L = X_C$, we can expand this to come up with an equation.

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC} \quad \text{multiply both side by } 2\pi fC$$

$$4\pi^2 f^2 LC = 1 \quad \text{divide both sides by } 4\pi^2 LC$$

$$f^2 = \frac{1}{4\pi^2 LC} \quad \text{square root both sides}$$

$$f_0 = \sqrt{\frac{1}{4\pi^2 LC}} \quad \text{simplify}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

In many instances, the frequency is labelled f_0.

In a series resonant circuit, the current rises to a maximum, as the only factor affecting it is the resistance.

The series resonant circuit is also sometimes called an '***acceptor circuit***'; as it accepts a high value of current at the resonant frequency.

Let's consider an example.

1). A 71 µF capacitor is connected in series with a 0.2 H inductor and a 24 Ω resistor. If the supply voltage is 120 V, what will be the resonant frequency, the total impedance at resonance, the current flowing at resonance and the phase angle?

This is the same example as before, but with an altered frequency.

a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.2\times 71\times 10^{-6}}} = \dfrac{1}{0.02368} = \underline{\underline{42.24Hz}}$

b) $X_L = 2\pi fL = 2\times\pi\times 42.24\times 0.2 = \underline{\underline{53.1\Omega}}$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\times\pi\times 42.24\times 71\times 10^{-6}} = \underline{\underline{53.1\Omega}}$

X_C and X_L are opposite

$\therefore$ Total $X = X_L - X_C = 53.1 - 53.1 = \underline{\underline{0\Omega}}$

c) $Z = \sqrt{R^2 + X_L^2} = \sqrt{24^2 + 0^2}$ $\qquad Z = R = \underline{\underline{24\Omega}}$

c) $I = \dfrac{U}{Z} = \dfrac{120}{24} = \underline{\underline{5A}}$

d). $\cos\phi = \dfrac{R}{Z} = \dfrac{24}{24} = \underline{\underline{1}}$

phase angle $\cos^{-1} 1 = \underline{\underline{0^\circ}}$

Quite simply, the resonant frequency has fallen to just above 42 Hz and at this point the only current that flows is due to the resistance within the circuit.

Exercise 6.

1) A 10 Ω resistor is connected in series with a 0.3 H inductor and a 20 µF capacitor. The supply voltage is 230 V and the frequency is 50 Hz. Determine:
   a) Impedance
   b) Current
   c) Phase angle
   d) Resonant frequency
   e) Current at resonant frequency.

2) A coil of wire has a resistance of 8 Ω and an inductance of 80 mH. It is connected to a 230 V 50 Hz supply. What value of capacitor would need to be connected to cancel out the effect of the inductor?

3) Fill in the gaps.

Inductance (H)	0.25		1.75		1
Capacitance (µF)	18	40		35	50
Resonant frequency (Hz)		60	100	250	

Now move on to the next session.

7: Phasor diagrams and voltage drops in a.c. circuits

In this session the student will:

- Practice creating phasor diagrams.
- Gain an understanding of voltage drops in series-connected a.c. circuits.

Over the last few sessions we have been looking at the relationship between resistance and reactance and the effects that they both have on an a.c. supply. We have introduced phasor diagrams, but we have not yet thought about what happens to the voltage dropped across the various elements.

You will remember that in the work done on series d.c. circuits, the volt drop across each resistance adds up the supply voltage.

The same cannot be said in a series a.c. circuit.

The voltages ***cannot*** be added together directly because they are 90° out of phase with each other. You must therefore apply Pythagoras to the voltages dropped across the inductor and the resistor.

Let's consider an example.

1). A pure inductor of inductance 15 mH and a resistor of resistance 10 Ω are connected across a 230 V, 50 Hz a.c. supply. Calculate:

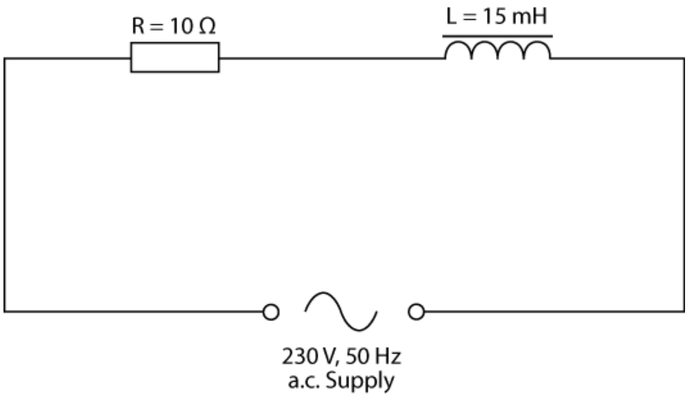
i) Reactance

ii) Impedance

iii) Current flow

iv) Volt drop across each element if they were separate

v) Phase angle.



You have done plenty of examples like this before.

$$i) \quad X_L = 2\pi f L$$

$$X_L = 2 \times \pi \times 50 \times 15 \times 10^{-3} = \underline{\underline{4.71\Omega}}$$

$$ii) \quad Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{10^2 + 4.71^2} = \underline{\underline{11.05\Omega}}$$

$$iii) \quad I = \frac{U}{Z} = \frac{230}{11.05} = \underline{\underline{20.81A}}$$

$$iv) \quad U_L = IX_L = 20.81 \times 4.71 = \underline{\underline{98V}}$$

$$U_R = IR = 20.81 \times 10 = \underline{\underline{208.1V}}$$

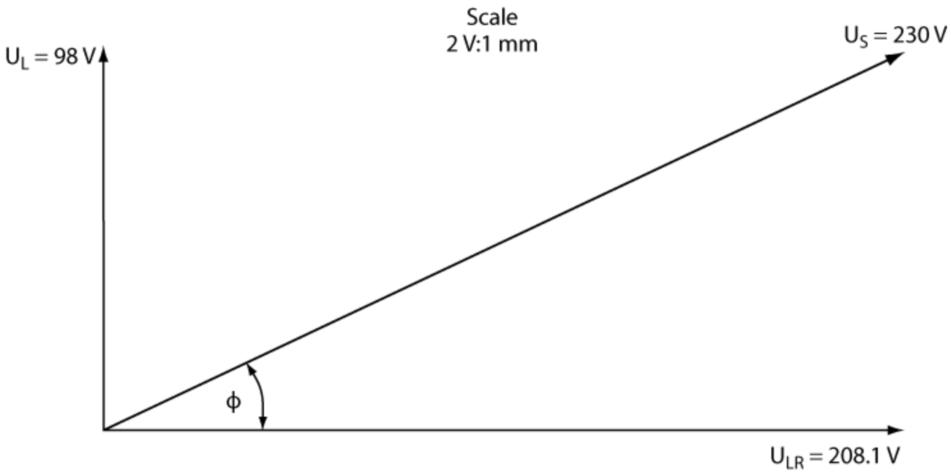
$$v) \quad \cos\phi = \frac{R}{Z} = \frac{10}{11.05} = \underline{\underline{0.905}}$$

$$\cos^{-1} 0.905 = \underline{\underline{25.18°}}$$

The point being here is that the two voltages don't simply add up to 230 V as they would do is we were dealing with pure resistors. So what do we do?

Pythagoras again!

Have a look at the diagram.

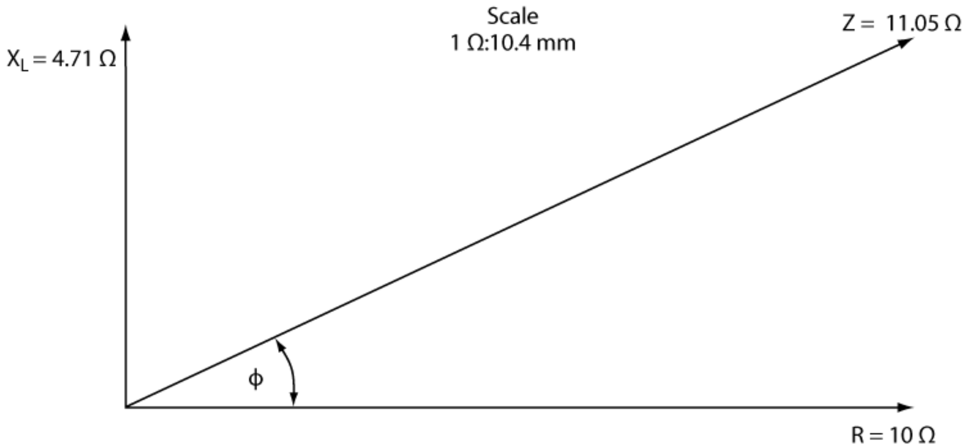


Here the voltages are measured in just the same way as you would for an ordinary series circuit. Have a look at the equation below.

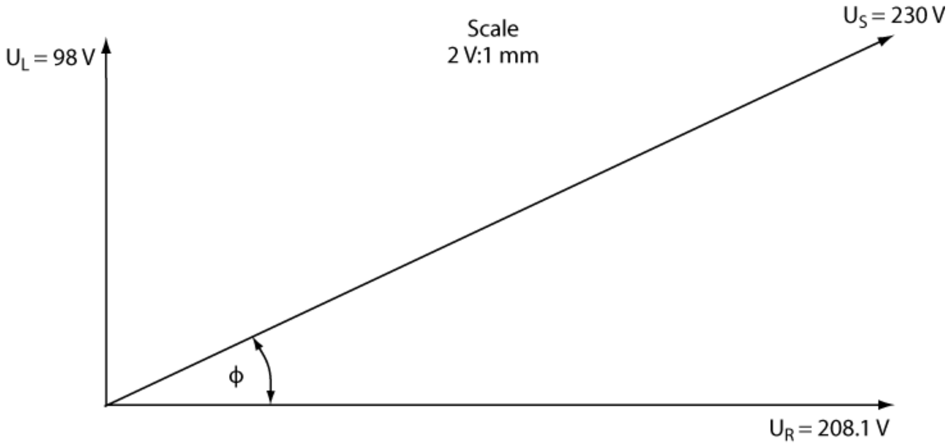
$$U_S^2 = U_R^2 + U_X^2$$
$$U_S = \sqrt{U_R^2 + U_X^2}$$
$$U_S = \sqrt{208.1^2 + 98^2}$$
$$U_S = \sqrt{52909} = \underline{\underline{230V}}$$

Notice that the two voltages, when added up using Pythagoras, make 230 V, which is, of course, our supply voltage.

Have a look at the two diagrams over the page.



Scales have changed but the shapes are the same



Here are a couple of phasor diagrams showing the voltages and the resistance/impedance.
When you consider both diagrams together, you can see that they have the same angle. Their values are different but the relationship between them isn't.

The scale of things is such however, that the phase angle is the same, irrespective of whether we are looking at impedance or voltage. Remember this!

Have a look back through the process before you look at the next example. This time I won't state what is happening, you should try to follow what is happening.

2). An inductor of inductance 0.32 H and a resistance of 200 Ω are connected in series. If the supply is 230 V, 50 Hz, what will be:

i) The inductive reactance

ii) The impedance

iii) The total current in the circuit

iv) The phase angle

v) The voltage dropped across the inductor and the resistor.

i) $X_L = 2\pi fL$

$X_L = 2 \times \pi \times 50 \times 0.318$

$X_L = \underline{\underline{100\Omega}}$

ii) $Z = \sqrt{R^2 + X_L^2}$

$Z = \sqrt{200^2 + 100^2}$

$Z = \sqrt{50000} = \underline{\underline{223.6\Omega}}$

iii) $I = \dfrac{U}{Z}$

$I = \dfrac{230}{223.6} = \underline{\underline{1.03\text{A}}}$

iv) $U_R = IR$

$U_R = 1.03 \times 200 = \underline{\underline{206\text{V}}}$

$U_{X_L} = IX_L$

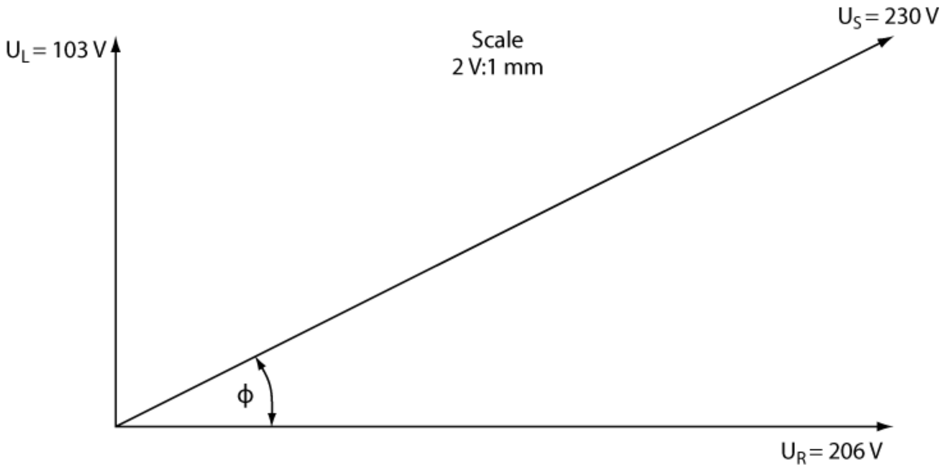
$U_{X_L} = 1.03 \times 100 = \underline{\underline{103\text{V}}}$

v) $\cos\phi = \dfrac{R}{Z} = \dfrac{200}{223.6} = 0.894$

$\phi = \cos^{-1} 0.894 = \underline{\underline{26.62°}}$

Remember the process. This example is just the same as the last one, with some of the numbers changed. Try getting the two voltages to add up to 230 V as shown on the previous page.

Again, it is worthwhile doing a couple of phasor diagrams. If this does nothing else, it helps you get into the habit.



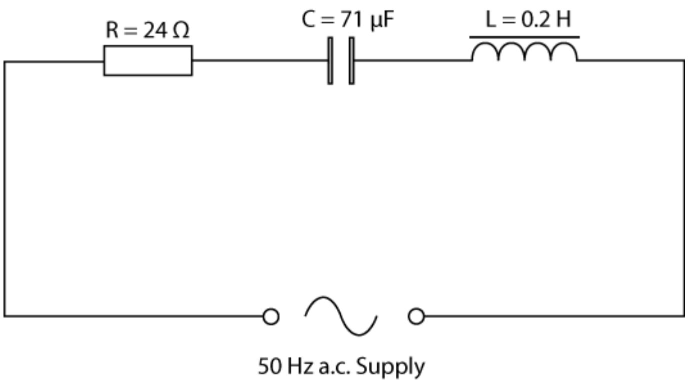
Again, notice that the phase angle is true for the voltages and the resistance, impedance, reactance phasor diagrams.

In a series circuit, the current is unchanging: this you already remember from the work that you have done at every previous level of study that you have done. Because of this, current is drawn in the reference position, or the horizontal, and so the voltage dropped across the inductor will be drawn upwards.

This makes the supply voltage drawn in such a position as to show the current lagging the voltage, assuming that the phasor diagram is moving in an anti-clockwise direction.

Let's consider an example containing all three components.

3). A 71 µF capacitor is connected in series with a 0.2 H inductor and a 24 Ω resistor. If the frequency is 50 Hz and the current is 4 A, what will be the voltage dropped across each component, and what will be the total supply voltage?



Remember to deal with each component in turn.

a) $X_L = 2\pi fL$

$X_L = 2 \times \pi \times 50 \times 0.2$

$X_L = \underline{\underline{62.83\Omega}}$

$X_C = \dfrac{1}{2\pi fC}$

$X_C = \dfrac{1}{2 \times \pi \times 50 \times 71 \times 10^{-6}}$

$X_C = \underline{\underline{44.83\Omega}}$

X_C and X_L are opposite

$\therefore$ Total $X = X_L - X_C = 62.83 - 44.83 = \underline{\underline{18\Omega}}$

b). $Z = \sqrt{R^2 + X_L^2}$

$Z = \sqrt{24^2 + 18^2}$

$Z = \sqrt{900} = \underline{\underline{30\Omega}}$

c). $U_S = IZ$

$U_S = 4 \times 30 = \underline{\underline{120\text{V}}}$

$U_R = IR$

$U_R = 4 \times 24 = \underline{\underline{96\text{V}}}$

$U_{X_C} = IX_C$

$U_{X_C} = 4 \times 44.83 = \underline{\underline{179.32\text{V}}}$

$U_{X_L} = IX_L$

$U_{X_L} = 4 \times 62.83 = \underline{\underline{251.32\text{V}}}$

Here you can see a strange number of features.

You will notice that the voltages that appear across the inductor and the capacitor are enormous, when compared to the supply voltage and the voltage dropped across the resistor.

The second point to note is that the voltage dropped across the inductor and the capacitor are acting in the opposite sense to each other. Effectively, because the overall circuit is inductive (there is more inductive reactance than capacitive reactance), then the voltage dropped across the inductor/capacitor part of the network can be added together.

$$U_{\text{Inductor/Capacitor}} = U_{X_L} - U_{X_C}$$

$$U = 251.32 - 179.32 = \underline{\underline{72\text{V}}}$$

This voltage shown is the overall effect of those two components being connected in a circuit together.

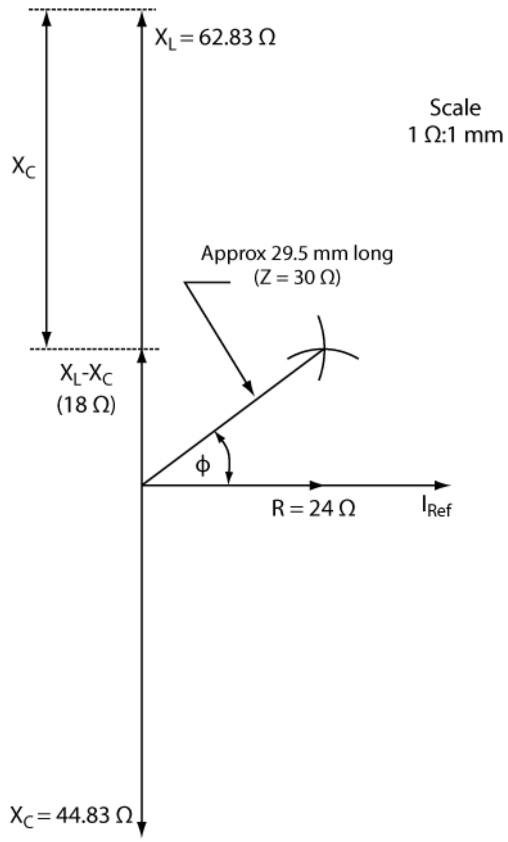
If we were to add the voltage dropped across the resistor and this overall voltage, then we would get the supply voltage.

$$U_S = \sqrt{U_R^2 + U_X^2}$$

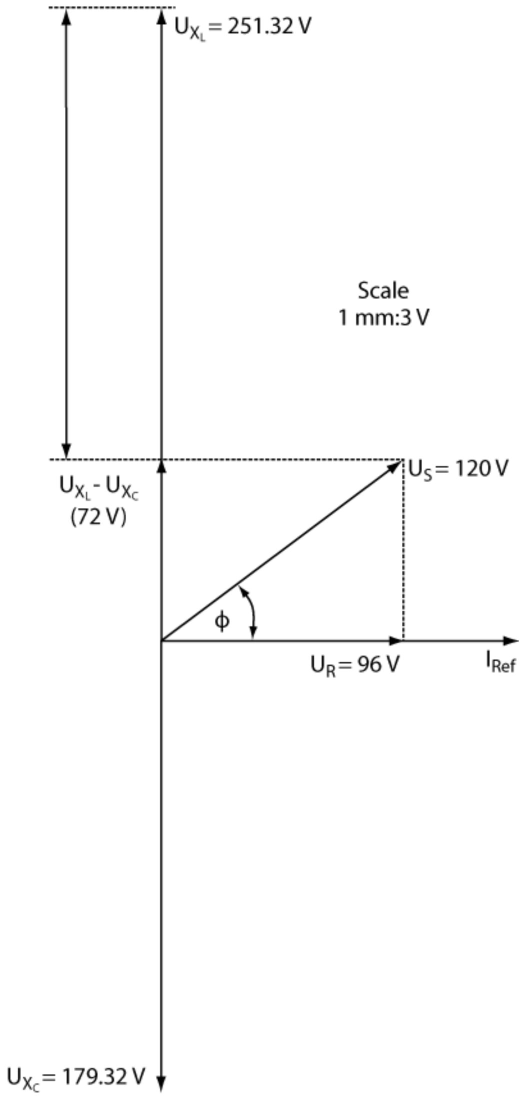
$$U_S = \sqrt{96^2 + 72^2}$$

$$U_S = \sqrt{14400} = \underline{\underline{120\text{V}}}$$

You can see that this enables all the values to drop out. Have a look at the completed phasor diagrams of both the resistance/reactance and also the voltages.



Note that the overall effect is to reduce the total reactance. The capacitive and inductive reactance values oppose each other and partially cancel each other out.



Here the overall effect can be seen of the voltage dropped across the individual components.

Notice that the phase angle, although not considered here, is still the same whether we are looking at voltage or resistance etc.

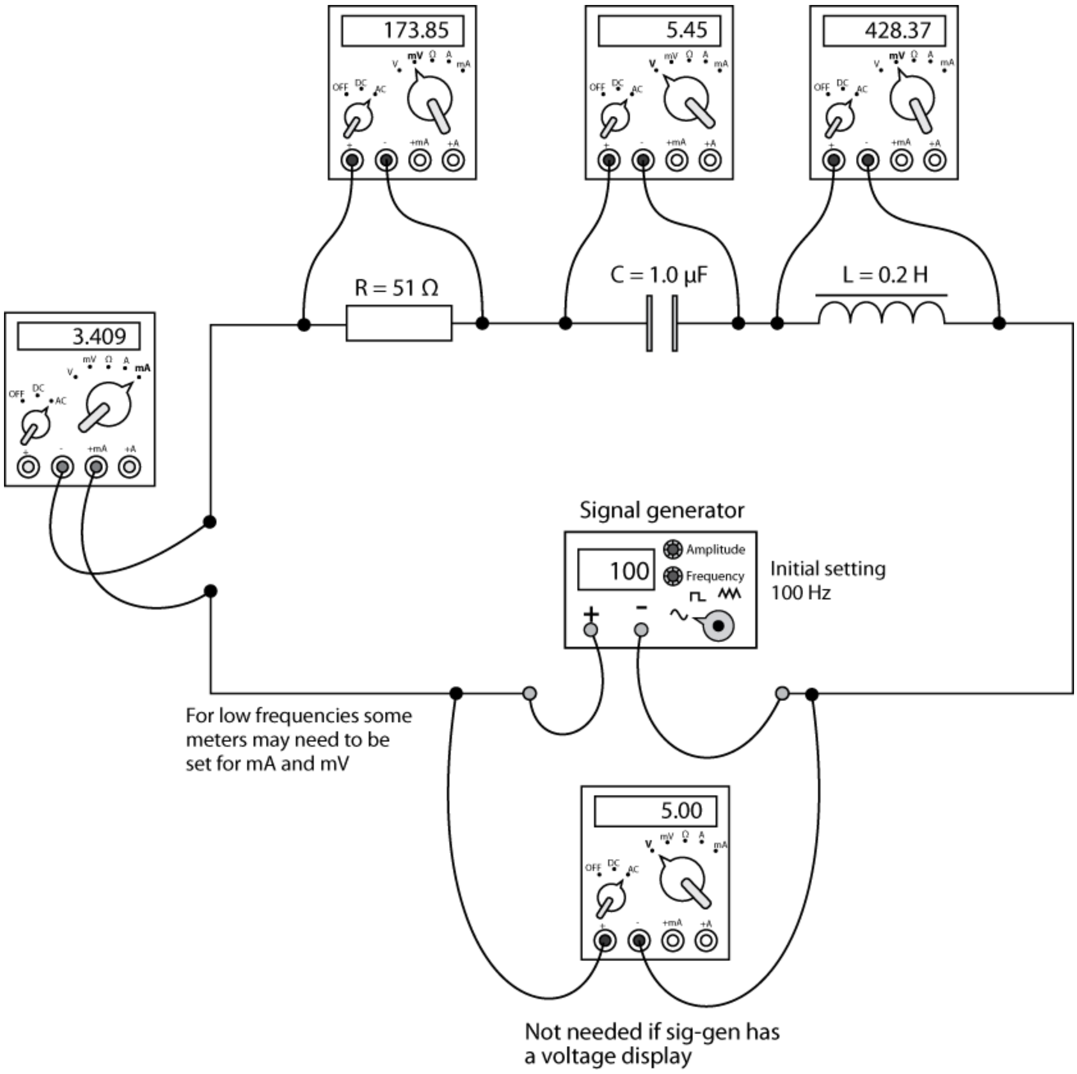
Exercise 7.

1) Set up a lab using a signal generator set at 5 V as a supply set to sine wave. Using the following components measure the voltage at a range of frequencies from 100 Hz to 1 kHz:

Inductor – 200 mH

Resistor – 51 Ω

Capacitor – 1 µF.



Measure the voltage across each component at each frequency.

This table is an example of how you might lay out your results. Initially I recommend that you measure the values at 10 Hz jumps in frequency. When you reach 100 Hz, then a 100 Hz jump until you reach 300 Hz. After 300 Hz make 25 Hz steps up to 400 Hz and then move back to making 100 Hz jumps until you reach 1 000 Hz.

Note: The table below is does not list all the frequency values that you will need to use, and therefore you will need to amend it accordingly.

Frequency	**U_S**	**U_R**	**U_L**	**U_C**	**I**
100					
200					
300					
400					
500					
600					
700					
800					
900					
1000					

Using the data you have gathered:

1) Draw a graph using frequency as the horizontal axis – being careful that the scale is constant – and all other values as the vertical axis values.
2) Determine from your graph and by calculation the resonant point.
3) Comment on why the two values may vary.
4) Using the 50 Hz, 500 Hz and 1 000 Hz values calculate what the expected values of volt drop and current should have been for the connected supply voltage.
5) Why do the numbers not match?
6) Draw a phasor diagram for each of the conditions listed in Q.4, using current as the reference (horizontal line).

8: Phasor diagrams and current flows in a.c. circuits

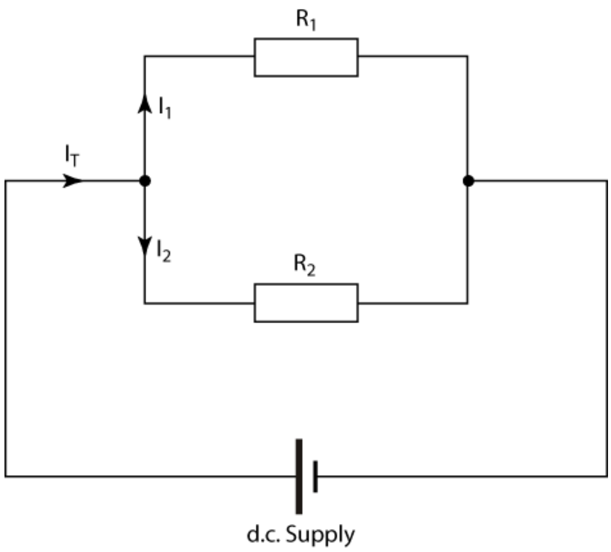
In this session the student will:

- Practice creating phasor diagrams.
- Gain an understanding of current flow in parallel-connected a.c. circuits.

We are not going to spend too much time looking at parallel circuits; however in some senses it is more important to get a grasp of the principles of parallel circuits than it is of series circuits.

You will remember, as it has been repeated so often, that in a ***series*** circuit the ***current remains constant***. The voltage is dropped across the individual components. These rules you should have learnt earlier.

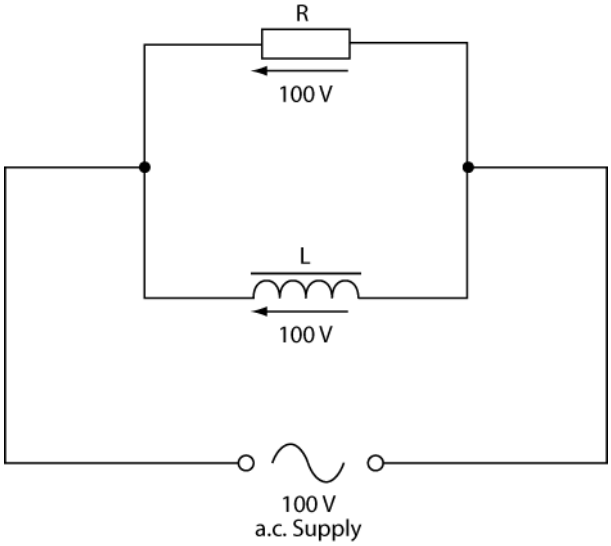
In ***parallel*** circuits, you should have learnt that it is the ***current that divides***: It is the ***voltage*** dropped across the parallel network is the ***same***. Have a look at the diagram below.



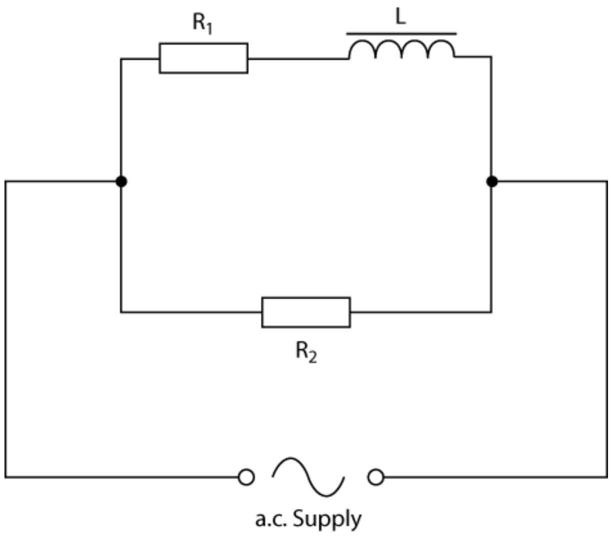
This is the classic d.c. condition.

In an a.c. circuit life is just the same. The voltage dropped across each leg is the same as for the other legs that are connected in parallel.

Below we you can see that with a 100 V supply and a resistor connected in parallel with an inductor, the voltage dropped across each is the same, 100 V.



And again with parallel circuits, when there is a combination of series and parallel arrangements then both sets of rules will apply at different moments.



Here you can see that there is a series arrangement in the top leg, although there is an overall parallel circuit as well. You would need to consider the top leg as a series circuit before you dealt with the parallel part of the network.

There is a slight difference to what we would normally expect from a d.c. parallel circuit and that is due to what happens to the current.

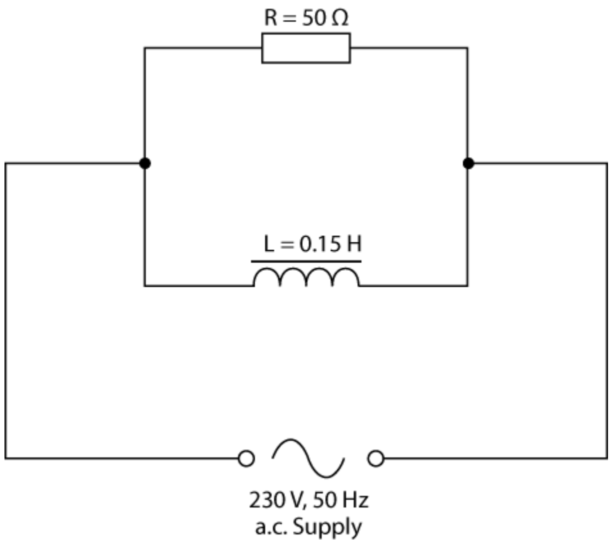
If the a.c. parallel circuit is made up of resistors, inductors and/or capacitors then the current will have been moved out of phase with the voltage. This means that we cannot add up the currents as we did with our d.c. parallel circuits. They are not in line with each other and so they cannot be added together. We have to go back to the tried and trusted adding up of phasor quantities.

As per normal, we'll work through a few examples to try to show what happens.

1). A 0.15 H inductor is connected in parallel with a 50 Ω resistor. They are supplied from a 230 V, 50 Hz supply. Calculate the:

   i) current in each leg of the circuit
   ii) total current
   iii) power factor and phase angle
   iv) impedance of the circuit.

It is worthwhile drawing a sketch.



You can see that this is quite a simple arrangement. Now try to follow the working out over the page.

$$i)\quad I_R = \frac{U}{R}$$

$$I_R = \frac{230}{50} = \underline{\underline{4.6\text{A}}}$$

$$X_L = 2\pi fL$$

$$X_L = 2\times\pi\times 50\times 0.15$$

$$X_L = \underline{\underline{47.12\Omega}}$$

$$I_{X_L} = \frac{U}{X_L}$$

$$I_{X_L} = \frac{230}{47.12} = \underline{\underline{4.88\text{A}}}$$

$$ii)\quad I_S = \sqrt{I_R^2 + I_X^2}$$

$$I_S = \sqrt{4.6^2 + 4.88^2}$$

$$I_S = \sqrt{44.34} = \underline{\underline{6.66\text{A}}}$$

$$iii)\quad \cos\phi = \frac{I_R}{I_S} = \frac{4.6}{6.66} = 0.69$$

$$\phi = \cos^{-1} 0.69 = \underline{\underline{46.37^\circ}}$$

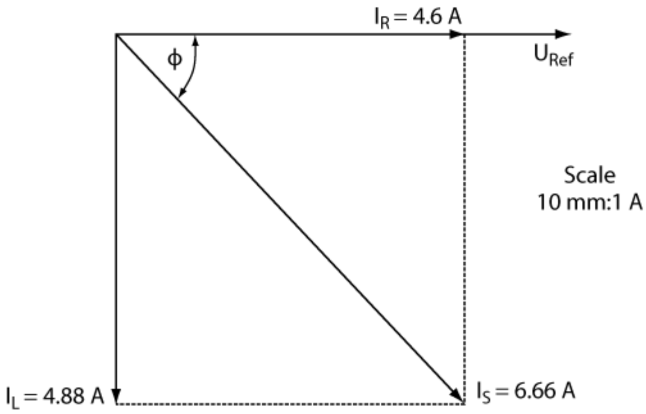
$$iv)\quad Z = \frac{U}{I}$$

$$Z = \frac{230}{6.66} = \underline{\underline{34.53\Omega}}$$

In this example, we are able to treat the supply voltage in the same way for both legs of the circuit.

The real key is recognising that the current in a resistor and in the inductor must be shifted 90° and so must be added up using Pythagoras' theorem. We must not forget that the currents in each leg have their own phase angle, and that must be accounted for.

You should also recognise how the power factor can be determined. Remember that the current in the resistor must be the effective adjacent side of a right-angled triangle and that the supply current provides the hypotenuse. Have a look at the diagram over the page.



In a parallel circuit, the voltage is drawn as the reference, as this does not change. With parallel circuits, the currents are divided down the different parallel legs.

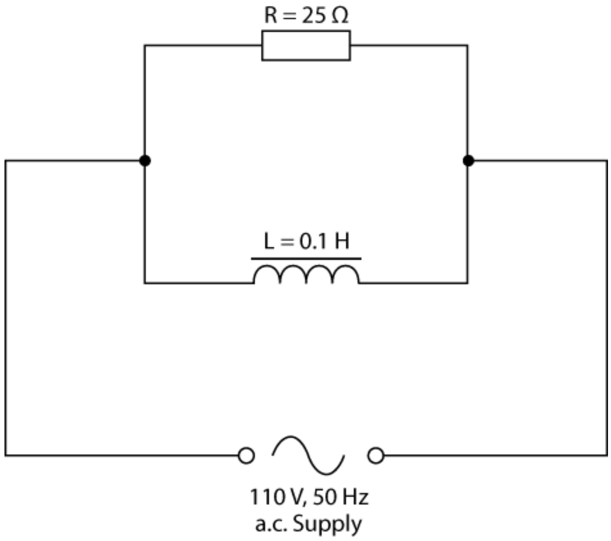
Note that the current, in a pure inductor, forms the vertical line and the current in the resistor is on the horizontal.

Try another example.

2). A 25 Ω resistor is connected in parallel with a 0.1 H inductor across a 110 V, 50 Hz supply. Determine:

i) The current in each leg
ii) The supply current
iii) The power factor and phase angle
iv) The total impedance.

Do the diagram again.



The working out is shown below.

$$i)\quad I_R = \frac{U}{R} = \frac{110}{25} = \underline{\underline{4.4A}}$$

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.1 = \underline{\underline{31.42\Omega}}$$

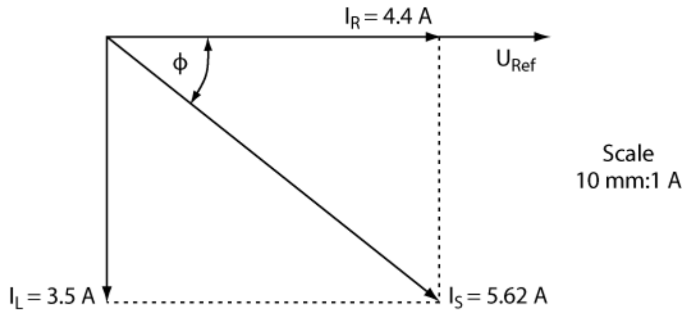
$$I_{X_L} = \frac{U}{X_L} = \frac{110}{31.42} = \underline{\underline{3.5A}}$$

$$ii)\quad I_S = \sqrt{I_R^2 + I_X^2} = \sqrt{4.4^2 + 3.5^2} = \sqrt{31.61} = \underline{\underline{5.62A}}$$

$$iii)\quad \cos\phi = \frac{I_R}{I_S} = \frac{4.4}{5.62} = 0.78 \quad \phi = \cos^{-1} 0.78 = \underline{\underline{38.5^\circ}}$$

$$iv)\quad Z = \frac{U}{I} = \frac{110}{5.62} = \underline{\underline{19.6\Omega}}$$

Exactly the same process has been followed as for the last example. Take your time and try to follow the process. Here is the phasor diagram.



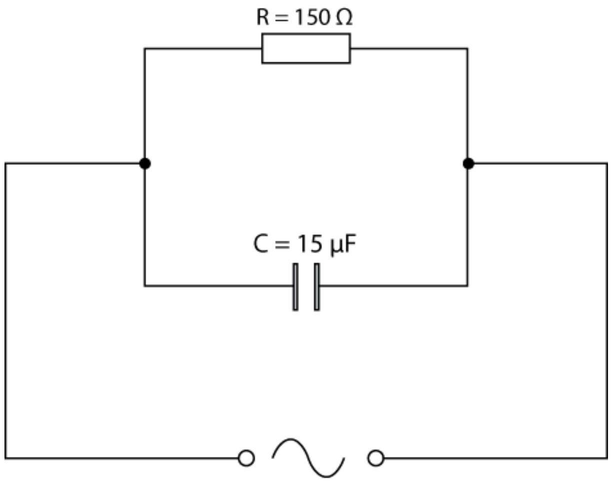
You should be able to check your own phasor diagram against the one shown above.

Now we'll move on. We've looked at resistors connected in parallel with inductors. Now we're going to look at resistors connected in parallel with capacitors. The process is virtually the same and one example should be enough.

3). A 150 Ω resistor is connected in parallel with a 15 µF capacitor. The supply is 230 V and 50 Hz. Determine the following:

i) Current in each leg
ii) Supply current
iii) Power factor and phase angle
iv) Impedance.

Remember to do a diagram.



The process is the same as that for resistors and inductors, so if you are not sure, have a look back at the examples already done.

$$i)\quad I_R = \frac{U}{R} = \frac{230}{150} = \underline{\underline{1.53\text{A}}}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times 50\times 15\times 10^{-6}} = \underline{\underline{212\Omega}}$$

$$I_{X_C} = \frac{U}{X_C} = \frac{230}{212} = \underline{\underline{1.1\text{A}}}$$

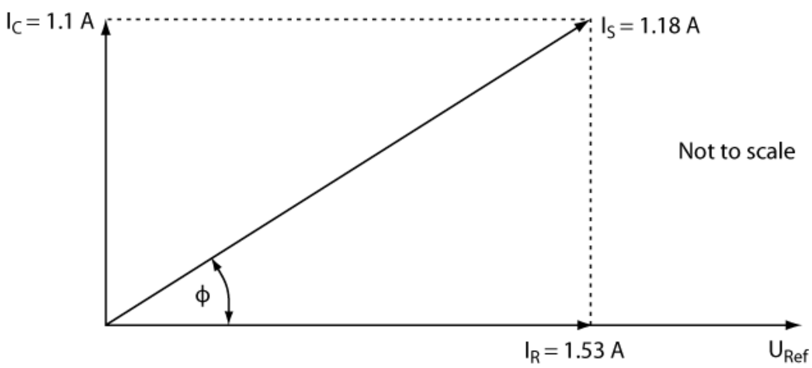
$$ii)\quad I_S = \sqrt{I_R^2 + I_X^2} = \sqrt{1.53^2 + 1.1^2} = \sqrt{3.55} = \underline{\underline{1.88\text{A}}}$$

$$iii)\quad \cos\phi = \frac{I_R}{I_S} = \frac{1.53}{1.88} = 0.81$$

$$\phi = \cos^{-1} 0.81 = \underline{\underline{35.9^\circ}}$$

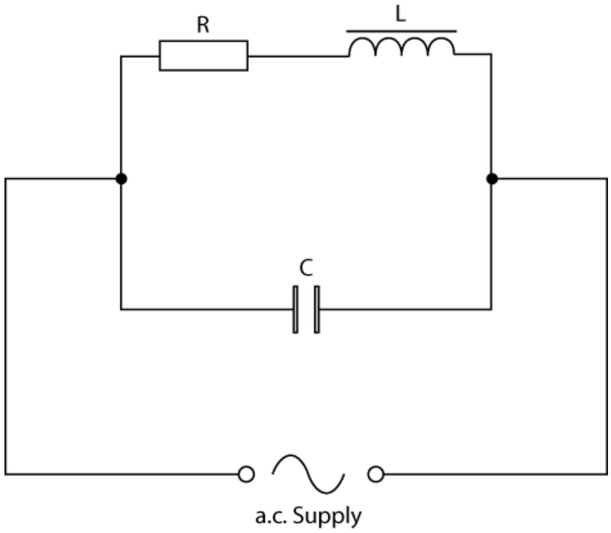
$$iv)\quad Z = \frac{U}{I} = \frac{230}{1.88} = \underline{\underline{122.3\Omega}}$$

Notice that the process is the same as that used for inductor/resistor network. The phasor diagram is shown below.



In this phasor diagram, you can see that the current in the capacitor has been drawn rising up from the horizontal (resistor current) line. This is in direct opposition to what happens when we have a resistor/inductor network.

Now that we have seen a capacitor/resistor network and an inductor/resistor network, it is time to look at a combination of these two. Have a look at the diagram below.



Here we have a resistor and inductor connected in series. Both of them are then connected in parallel with a capacitor.

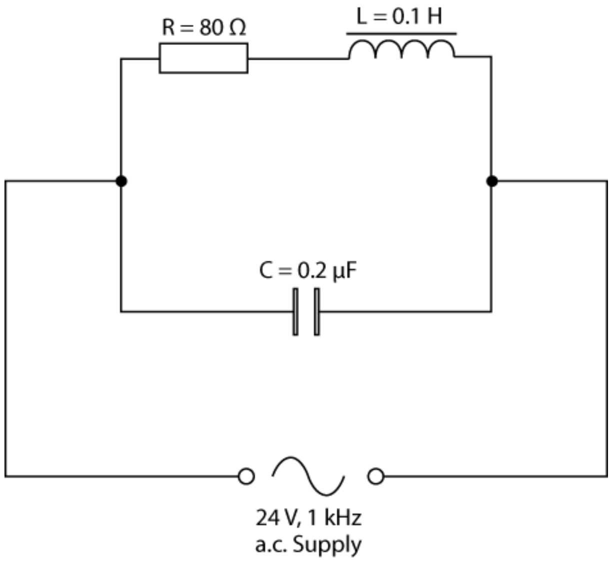
This arrangement could not be more practical as, for example, this type of circuit is produced by a single-phase capacitor start motor or a discharge fitting with a power factor correction capacitor.

We'll work through a couple of examples so that you can get a feel for them.

4). A 0.2 µF capacitor is connected in parallel with a coil. The coil has a resistance of 80 Ω and an inductance of 0.1 H. The supply to the circuit is 24 V at a frequency of 1 kHz. Determine:

i) Impedance of the coil
ii) Current in each leg of the circuit
iii) Supply current
iv) Power factor and phase angle.

Have a look at the diagram.



Here you can see how the circuit is arranged. If you follow the order in which the question is asked you should get some clues as to what you should do.

Remember however to be methodical and to deal with each leg in turn.

i) Coil $X_L = 2\pi fL = 2\times\pi\times1000\times0.1 = \underline{\underline{628.3\Omega}}$

$$Z = \sqrt{R^2 + X^2} = \sqrt{80^2 + 628.3^2} = \sqrt{401161} = \underline{\underline{633.4\Omega}}$$

ii) $I_{\text{Coil}} = \dfrac{U}{Z} = \dfrac{24}{633.4} = \underline{\underline{37.9\text{mA}}}$

Coil p.f. $\cos\phi = \dfrac{R}{Z} = \dfrac{80}{633.4} = 0.126$

$$\phi = \cos^{-1} 0.126 = \underline{\underline{82.74^\circ}}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times1000\times0.2\times10^{-6}} = \underline{\underline{795.7\Omega}}$$

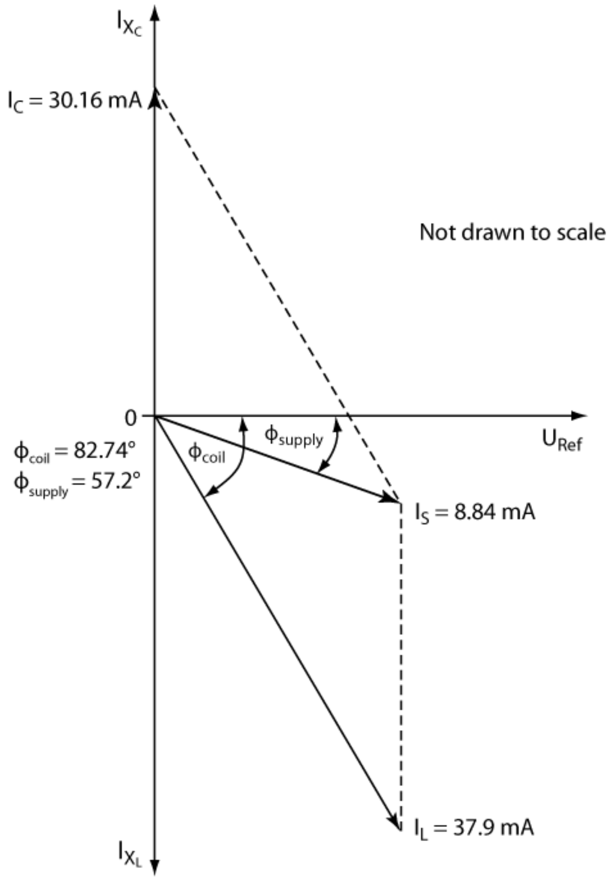
$$I_{\text{Cap}} = \frac{U}{Z} = \frac{24}{795.7} = \underline{\underline{30.16\text{mA}}}$$

$\phi = 90^\circ$ leading for a capacitor

So far so good. However, if you look at the phasor diagram over the page, you will see that the current in the coil and the current in the capacitor are not 90° apart. This means that we can't yet use our old friend Pythagoras to add up the currents.

There are two possible ways of dealing with the problem of adding up the values of current. The easiest way by far is to draw your phasor diagram to scale and then begin to draw some lines.

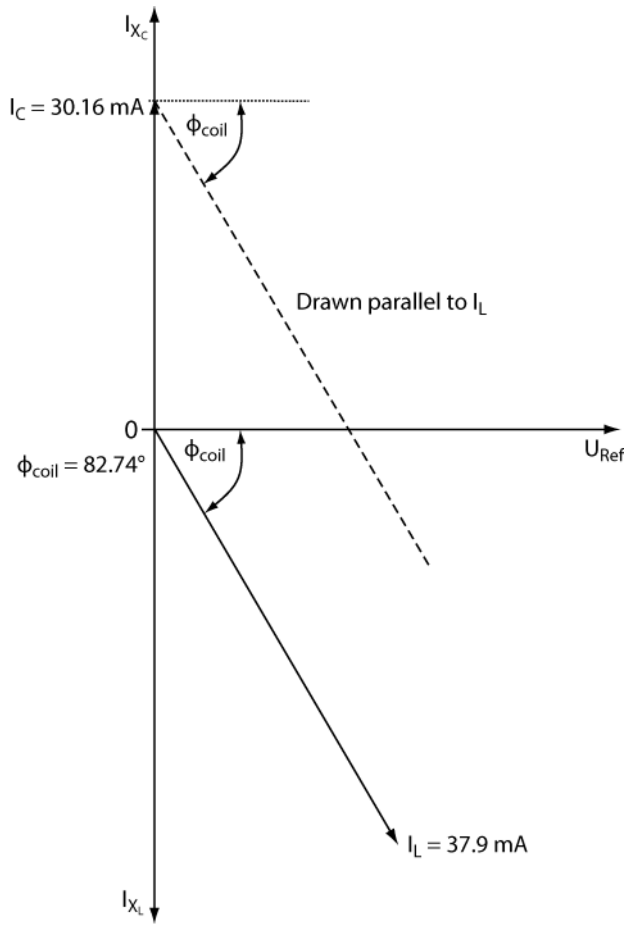
The diagram below answers parts iii) and iv) of the question.



How did we manage to create this diagram? If you follow the diagrams over the next few pages you will see how this final arrangement has been created.

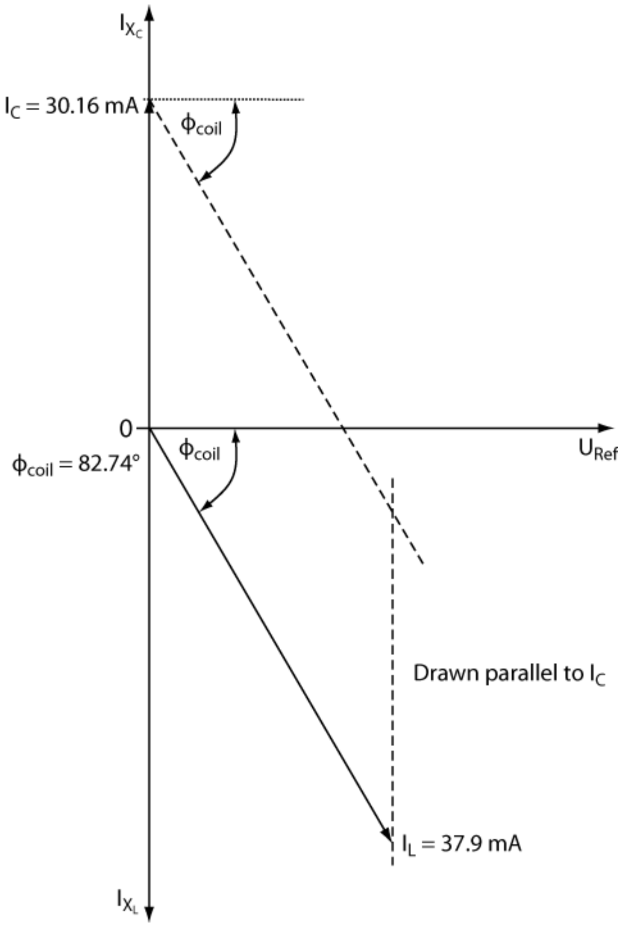
Follow the process over the page.

From the end of the line describing the current in the capacitor, draw a line parallel with the line describing the current in the coil.



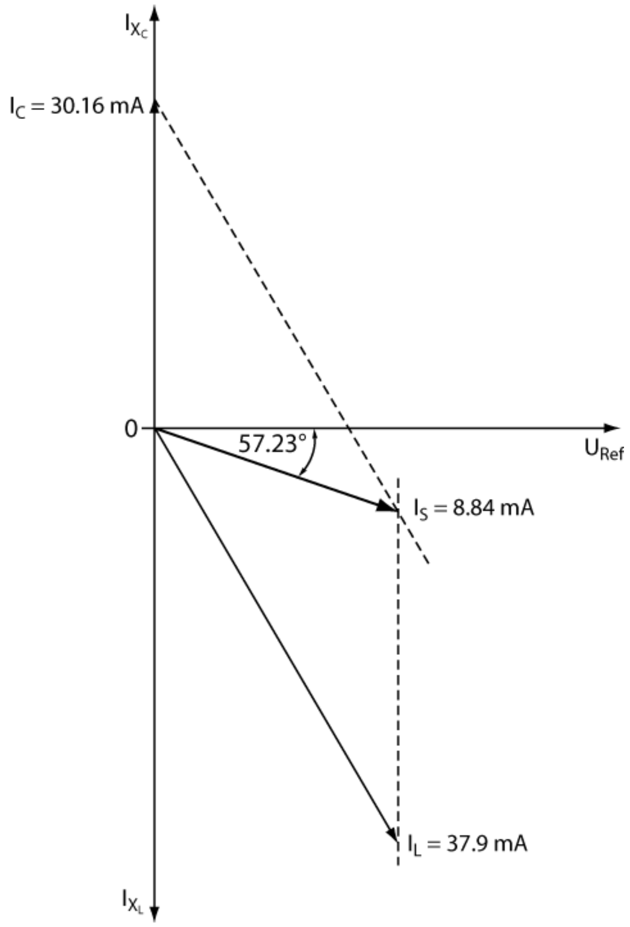
You can follow this in the diagram above by following the dotted line. It doesn't matter how long this line is. It is better to make the line too long rather than too short.

From the end of the line describing the current in the coil, draw a line parallel with the current in the capacitor line.



Again, follow the diagram using the dotted line. They should cross.

In this diagram, you can see that a line has been drawn from the start point to the point where the lines cross.



It is this line that shows what the supply current is; and the angle that this line forms with the horizontal is the phase angle. Pressing the cosine button on the calculator when this number has been tapped in will give you the power factor.

You should find that the total current (supply) will be 8.85 mA and the phase angle will have reduced to 57.23˚.

The second way of determining this information is mathematically. The mathematical method is more accurate, however at this level of study unnecessary.

Try another example.

5). A coil has a resistance of 65 Ω and an inductance of 0.318 H. If a capacitor of capacitance 35 μF is connected in parallel with the coil, determine the following if the supply voltage is 200 V and the frequency is 50 Hz:

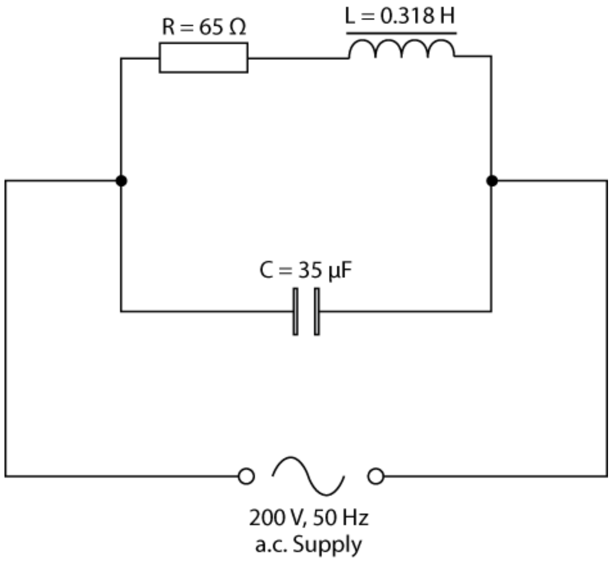
i) Impedance of the coil

ii) Current in each leg of the circuit

iii) Supply current

iv) Power factor and phase angle.

Draw a sketch of the circuit.



The working out is shown below. We are going to determine the power factor and the supply current via the use of phasor diagrams.

i) Coil $X_L = 2\pi fL$

$X_L = 2 \times \pi \times 50 \times 0.318 = \underline{\underline{100\Omega}}$

$Z = \sqrt{R^2 + X^2} = \sqrt{65^2 + 100^2} = \sqrt{14225} = \underline{\underline{119.3\Omega}}$

ii) $I_{\text{Coil}} = \frac{U}{Z} = \frac{200}{119.3} = \underline{\underline{1.68\text{A}}}$

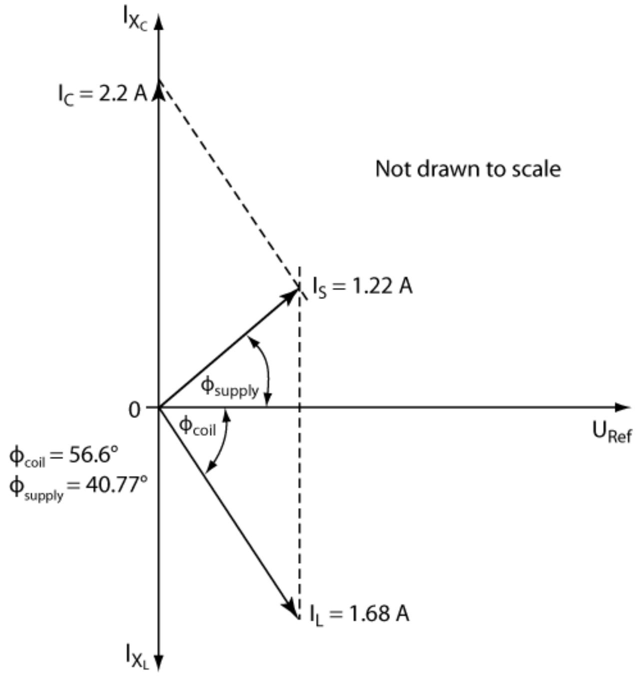
Coil p.f. $\cos\phi = \frac{R}{Z} = \frac{65}{119.3} = 0.55 \quad \phi = \cos^{-1} 0.55 = \underline{\underline{56.6^\circ}}$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 35 \times 10^{-6}} = \underline{\underline{91\Omega}}$

$I_{\text{Cap}} = \frac{U}{Z} = \frac{200}{91} = \underline{\underline{2.2\text{A}}}$

$\phi = 90^\circ$ leading for a capacitor

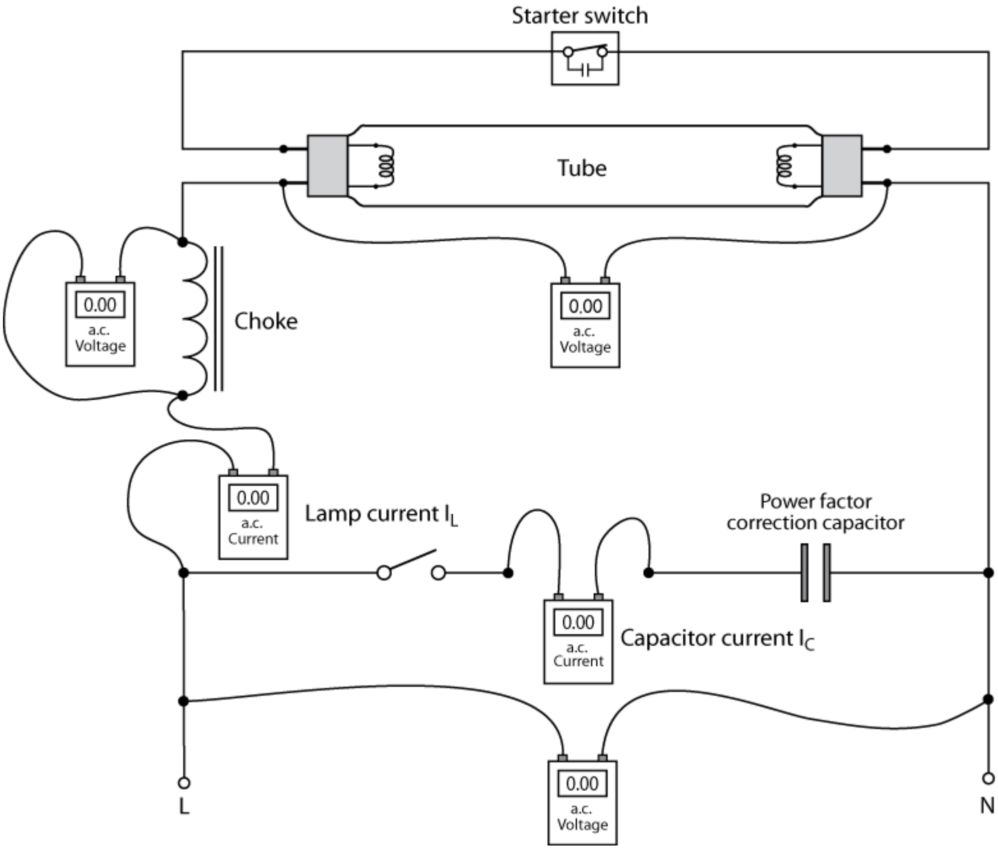
The phasor diagram is shown below. Remember to draw this to scale.



This is really the simplest way of dealing with this type of problem, and we can see that parts iii) and iv) of the problem are now solved. The total current drawn from the supply is 1.22 A, and the power factor and phase angle are 0.757 leading and 40.77° respectively.

Exercise 8.

1) Connect together a simple switch-start fluorescent lighting circuit.



i) Set up the circuit and take all readings of voltage and current with the switch in the capacitor leg open.

ii) Repeat (i) with the switch in the capacitor leg made.

iii) What do you notice about the changes in values of current?

iv) Draw a phasor diagram of what you think is happening.

v) What errors do you think are inherent in the experiment?

9: Power and power factor in a.c. circuits

In this session the student will:

- Gain an understanding of true, apparent and reactive power.
- Gain an understanding of power factor in a.c. circuits.

You will have come across the terms energy, power and efficiency before. It is important, and it is therefore useful to just take a few moments to re-acquaint yourself with some of the basic principles.

Work/Energy

Work is done whenever an object is moved through a distance. Work relates the force applied to an object, to the distance that the object moves.

In purely mechanical terms, work is measured in joules. This is the same unit as energy is measured in and the two can be quite easily interchanged.

Energy is usually given off in the form of heat. It doesn't matter whether we are looking at mechanical work or electrical work, the units are the same and interchangeable.

Power

Power is the rate at which work is done, or the rate at which energy is dissipated. There are two mechanical terms used to define power. One is used for power operating in a straight line, the other for power delivered using rotation.

$$\text{Power} = \frac{W}{t} = \frac{Fd}{t}$$

$$\text{Power} = 2\pi nT \quad \left[n = \text{speed } (\text{r/s}) \quad T = \text{torque } (\text{Nm}) \right]$$

These are fine, and you should certainly be able to use the second of the terms, it is particularly useful when dealing with motors and generators. However, we are dealing with electrical power.

To determine electrical power there are many formulae that can be used, and they all relate current, voltage and resistance.

Have a look at the table below. These are the three power formulae with their variants.

$P = IU$	$I = \frac{P}{U}$	$U = \frac{P}{I}$
$P = I^2R$	$I = \sqrt{\frac{P}{R}}$	$R = \frac{P}{I^2}$
$P = \frac{U^2}{R}$	$U = \sqrt{PR}$	$R = \frac{U^2}{P}$

When we consider power in an a.c. system we have a little more to consider than just resistance however. We already know that in an a.c. circuit there is not only resistance, but also reactance, and the overall effect is to provide us with impedance.

Power in an a.c. system

We have already seen that in a normal a.c. circuit there are three distinct factors.

- **Resistance (R) and depends solely on the length, area and type of material that makes up the load.**
- **Reactance (X). Reactance is a measure of the inductive or capacitive effect of the circuit, and where resistance opposes current flow the reactance opposes current change. Both are however measured in ohms (Ω).**
- **Impedance (Z) is a combination of resistance and reactance and is also measured in ohms.**

The total supply current takes into account the combination of resistance and reactance and uses impedance as the means of determining it.

$$I_S = \frac{U}{Z}$$

$$I_R = \frac{U}{R}$$

$$I_X = \frac{U}{X}$$

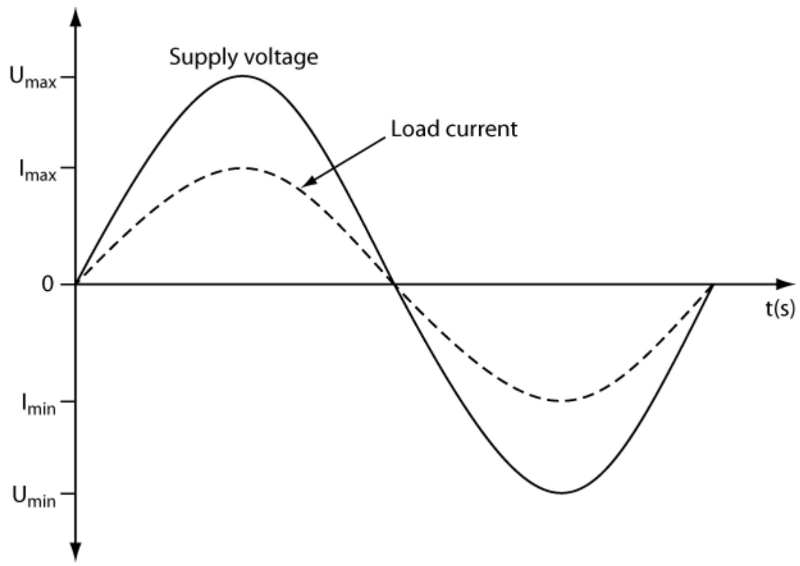
Now these three equations all provide a value of current and they may all be appropriate in any one particular instance.

So which one do we use when we are trying to determine the power consumed? There are in effect three ways in which we look at power in an a.c. circuit. The three types of power are called:

- **True Power**
- **Apparent Power**
- **Reactive Power.**

True Power

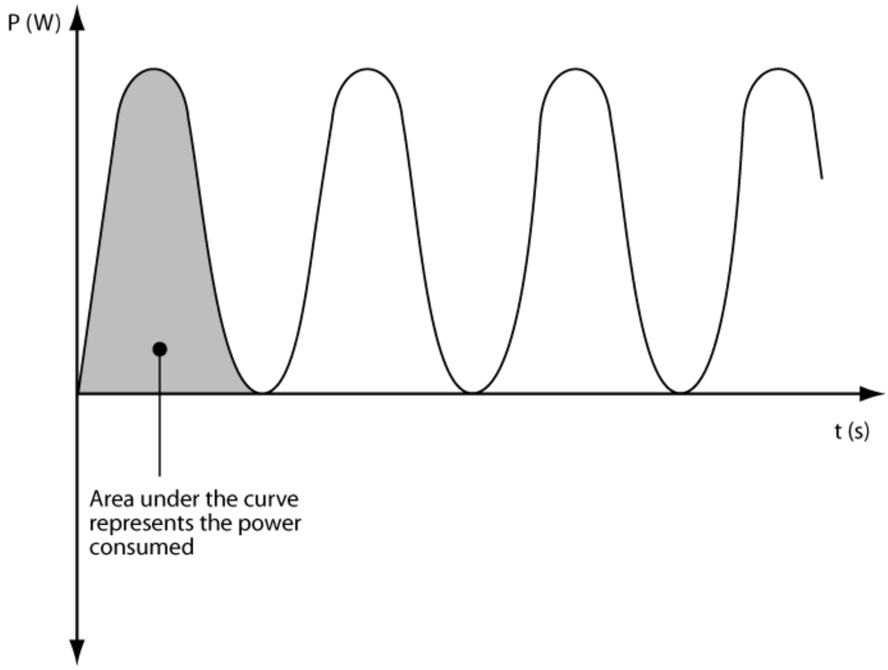
True power is a measure of the power used when considering the resistance value only. It is measured in watts (W) or kilowatts (kW). Consider the sine wave below.



Here we see the voltage and current sine waves in a purely resistive circuit. You can see that they are 'in phase' with each other. If we then multiply them together, as we would when we are trying to determine power $(P = IU)$, then we get a sine wave.

This time however there is no negative part to the cycle. This makes sense as when two negative values are multiplied together then we get a positive value.

The area under the curve of the graph shows the power absorbed by the circuit.



The calculation of true power is the same as for d.c. circuits and is often called the I^2R power or loss.

You can think of true power as useful power.

Reactive Power

Reactive power is often called wattless power. It is measured in 'reactive volt-amperes', 'VAr's' or 'kVAr's'.

Reactive power is the measure of the power in terms of either, an inductor or capacitor, and has the symbol 'Q'. Have a look at Figure 58.

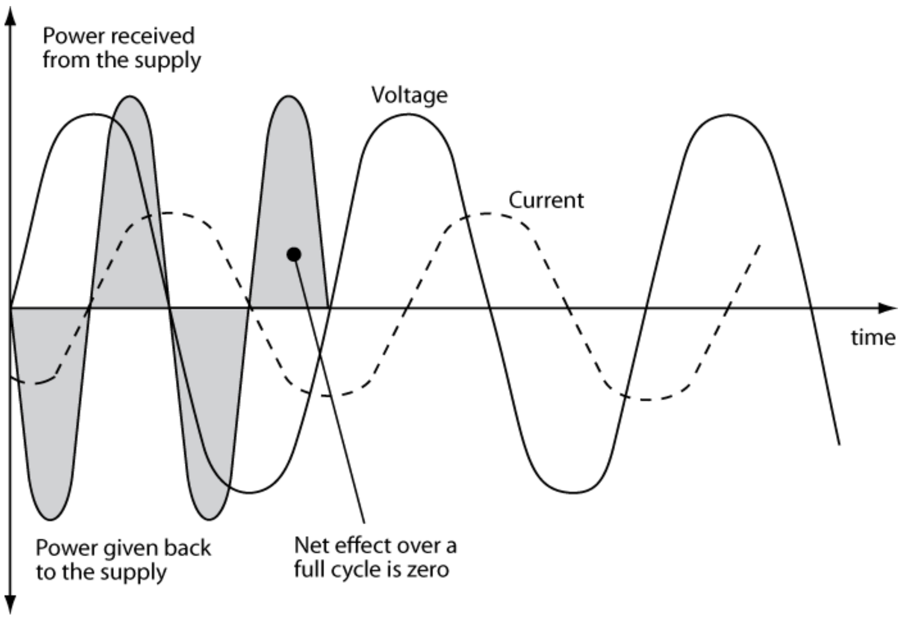


Figure 1 Power diagram in a purely inductive a.c. circuit

Here, the supply voltage is 90º out of phase with the current, assuming that we have a pure inductor or capacitor. It doesn't matter now whether we are looking at a purely capacitive or inductive circuit, the consequences are the same.

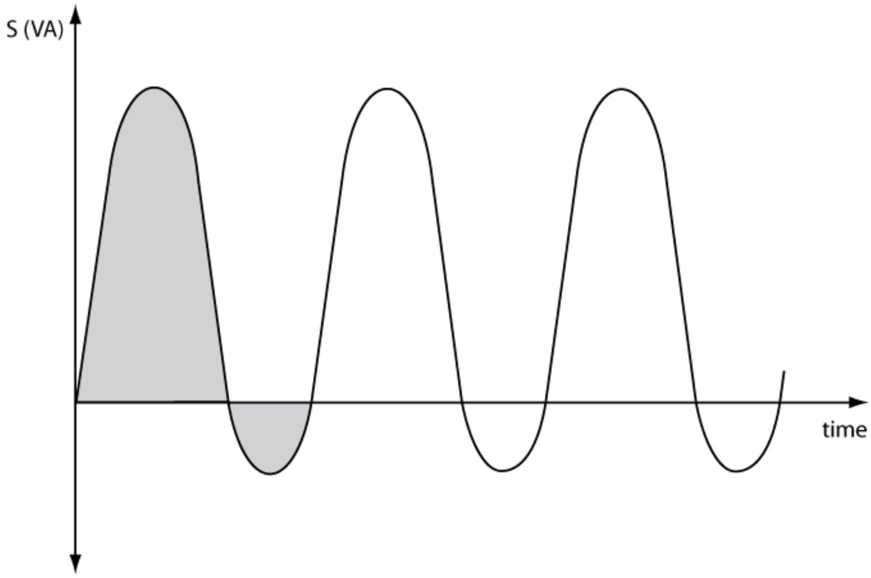
If the voltage and current are now multiplied together, the power alternates between the positive and negative half cycle. This is because the inductor or capacitor is storing energy on one half-cycle and then returning it to the supply on the next half cycle. The overall effect is that the 'pure' inductor or capacitor uses no power. It is, as its name suggests, *wattless*!

The pure resistor has I^2R losses and the pure inductor/capacitor has I^2X losses (which are no losses at all). We know however, that we will rarely get a pure resistor or inductor/capacitor.

Apparent Power

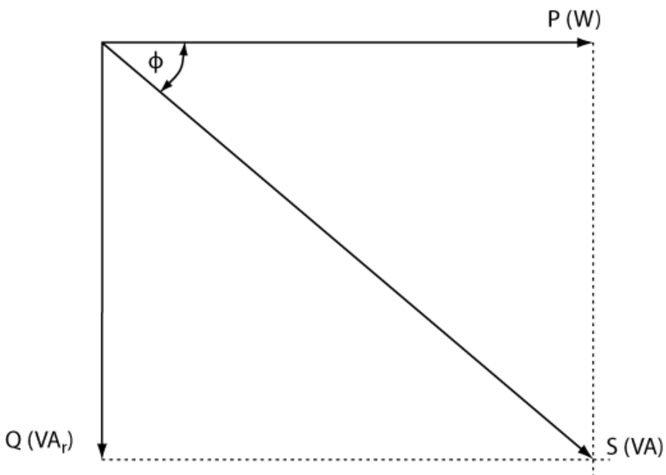
Apparent power is a measure of the combined effects of resistive and reactive power. Because there is no pure resistor, inductor or capacitor, the overall power demand is always going to be a combination of the two.

Apparent power is measured in 'volt-amperes', 'VA', or 'kVA', and has the symbol, 'S'. Have a look below.

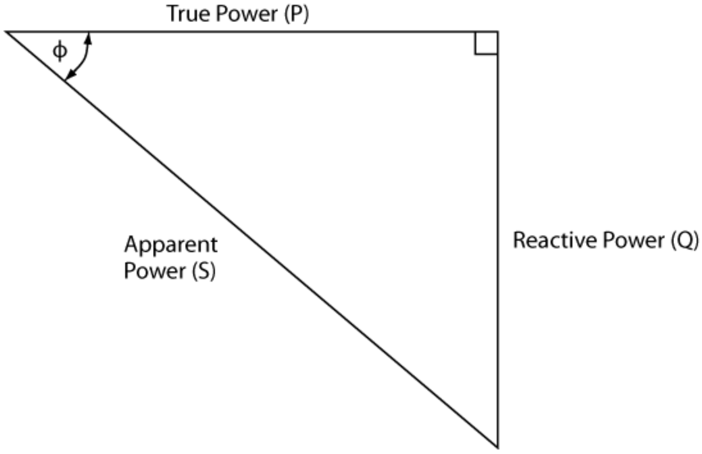


Here the power wave is mostly above the horizontal line, and so most is consumed in the resistive part of the circuit. However, a significant amount sits under the line. This part is returned to the supply, and is due to the reactance of the circuit.

We can combine all these three values into a power triangle. You will have seen a number of these triangles before.



Here the true power (P) is represented by the horizontal line. The vertical line represents the reactive power (Q). The line that joins the two represents the apparent power (S). This type of diagram can be drawn either as you can see above or below.



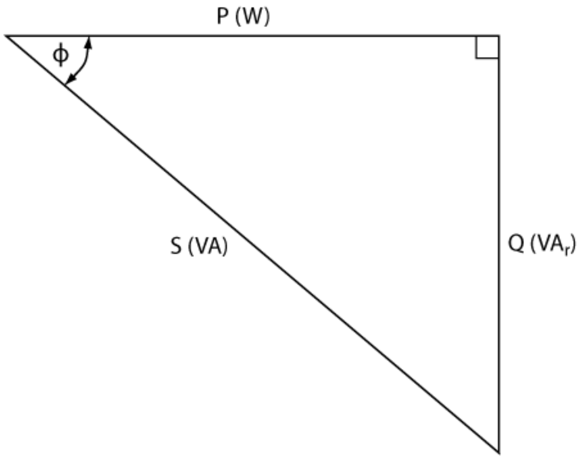
What you will see is a classic right-angled triangle.

Two relationships should be understood when we relate apparent power to reactive and true power.

As you will remember, Pythagoras' theorem comes into play here. So:

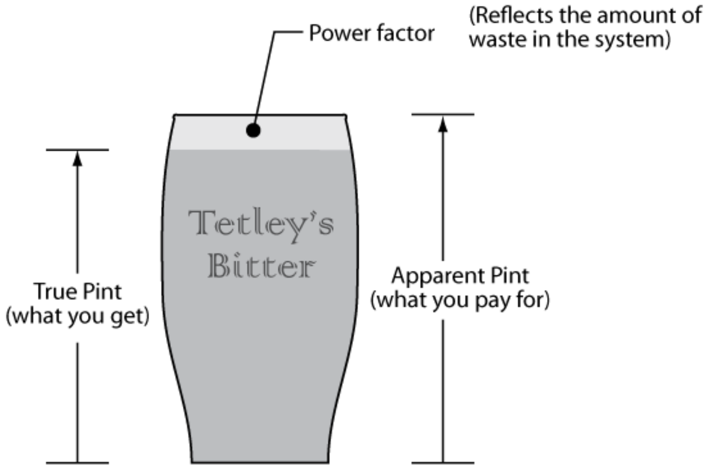
$$S = UI \quad [\text{VA}]$$
$$P = UI\cos\phi \quad [\text{W}]$$
$$Q = UI\sin\phi \quad [\text{VAr}]$$

These three relationships relate the three sides of the triangle together.



The angle (ϕ) is the angle formed by the apparent power and the true power.

There is one term that we have used quite often now without giving it much of an explanation and that is '***power factor***'.



This analogy shows that the beer (the part that is useful) is slightly less than the maximum that the glass can hold. The head (power factor) therefore reflects the waste in the system – or in this instance lack of beer.

The power factor of a system can be defined in a number of ways. It is the relationship between true and apparent power. It is also the relationship between resistance and impedance.

It is, in effect, a measure of how far the current and the voltage of a system are 'out of phase' with each other.

We'll just take a moment to consider what all this means.

1). A transformer is able to deliver 200 A at a voltage of 1000 V. The maximum apparent power that is available is the product of the voltage and current. So:

$$S = UI = 200 \times 1000 = \underline{\underline{200000\text{VA}}} = \underline{\underline{200\text{kVA}}}$$

This is the maximum that can be delivered to a system.

We'll now consider a series of currents and voltages that are in phase with each other and progressively out of phase with each other.

Let us consider what happens when different loads are connected to the transformer. Let these loads have different power factors.

$$P = UI\cos\phi$$

$$0^\circ \quad P = 200 \times 1000 \times \cos 0 = 200 \times 1000 \times 1 = \underline{\underline{200\text{kW}}}$$

$$20^\circ \quad P = 200 \times 1000 \times \cos 20 = 200 \times 1000 \times 0.9397 = \underline{\underline{187.94\text{kW}}}$$

$$40^\circ \quad P = 200 \times 1000 \times \cos 40 = 200 \times 1000 \times 07660 = \underline{\underline{153.21\text{kW}}}$$

$$60^\circ \quad P = 200 \times 1000 \times \cos 60 = 200 \times 1000 \times 0.5 = \underline{\underline{100\text{kW}}}$$

$$90^\circ \quad P = 200 \times 1000 \times \cos 90 = 200 \times 1000 \times 0 = \underline{\underline{0\text{kW}}}$$

You can see here that with an ever-increasing difference in phase angle between voltage and current, we get a steady reduction in the true power, even though the transformer is still delivering the same power.

It is for this reason that electrical plant is rated in kVA instead of kW, the designers do not know what load is going to be connected.

When we look at the series of problems above we can see a series of numbers beginning with 1 and going through 0.9397, 0.7660, 0.5 and 0. These values are the cosines of the particular angles and are termed the 'power factor'.

In the first example, we have angle of 0° this gives a power factor of 1 or unity. (cos0° = 1)
At 20º we have a power factor of 0.9397;
At 40º we have a power factor of 0.7660;
At 60º we have a power factor of 0.5;
And at an angle of 90º we have a power factor of 0.

You can see therefore that it is better to try to have a power factor closer to unity than zero.

As electricians, you may spend some time looking at how this power factor can be improved as it provides a number of very definite benefits. These are:

- by increasing the power factor to unity the apparent power is brought closer to the true power and the current reduces
- as the current reduces so do the cable sizes and the switchgear
- as the current falls then so does the demand. As the demand falls then so does the electric bill.

There are three methods of improving the power factor of an installation.

As most loads are inductive by nature, and as a capacitor has the opposite effect to that of an inductor, connecting capacitors across the load will improve the power factor.

This is often seen in discharge lighting circuits and small motors.

The second way of improving the overall power factor is to connect capacitors across the total supply at the intake position. This is called ‘bulk correction’.

The third method is to use a synchronous motor. (A synchronous motor can be run with a leading power factor and this will improve a lagging power factor). This is the most expensive way of dealing with the problem however and is rarely used except in large industries.

Exercise 9.

1) An a.c. single-phase circuit containing resistance of 35 Ω, inductive reactance of 25 Ω and capacitive reactance of 12 Ω. If the circuit is connected across a 230 V, 50 Hz supply, determine:

   i) Power factor
   ii) True power
   iii) Reactive power
   iv) Apparent power.

2) What would happen in Q.1 if the capacitive reactance were to increase to 19 Ω?

3) Using the internet, investigate the types of power factor correction capacitors that are available for bulk correction.

4) Draw a power triangle for both Q.1 and Q.2.

10: Three phase star and delta connections

In this session the student will:

- Gain an understanding of star and delta connected three phase supplies.

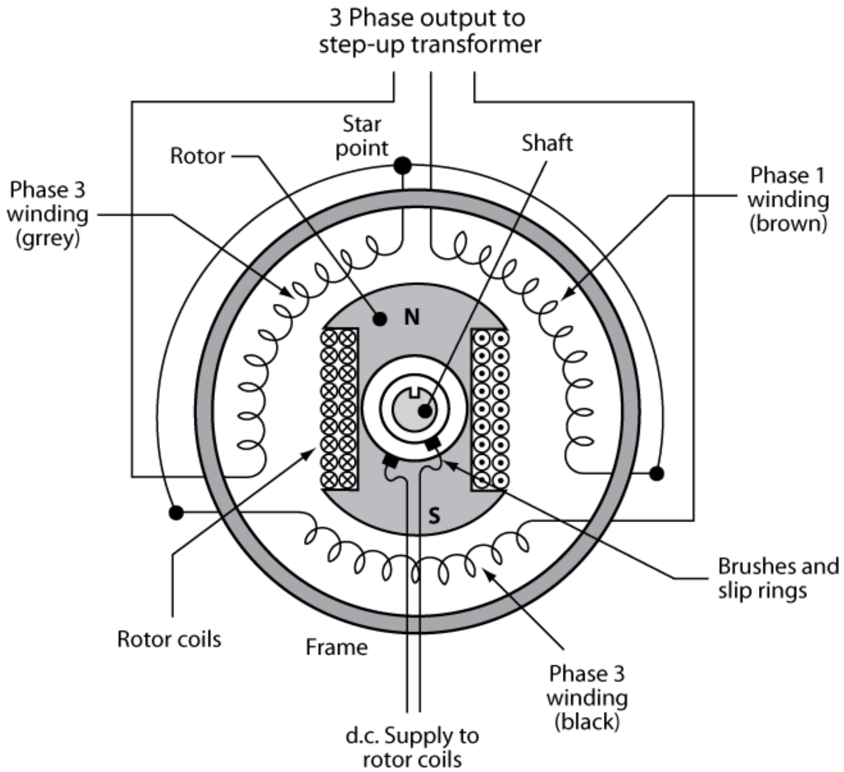
This session takes a look at the two ways in which a three-phase supply can be connected. No doubt, you are already familiar with the concept that a single-phase supply has a nominal voltage of 230 V and that a three-phase supply will typically have a nominal voltage of 400 V. Where does this value come from? How many ways are there of connecting three wires to a load and what is the relationship between different values of voltage and current?

A three-phase supply is a natural function of the generation process. A generator consists of three distinct electrical windings.

When the generator is turned within a magnetic field these windings, which are placed 120º away from each other, produce an emf

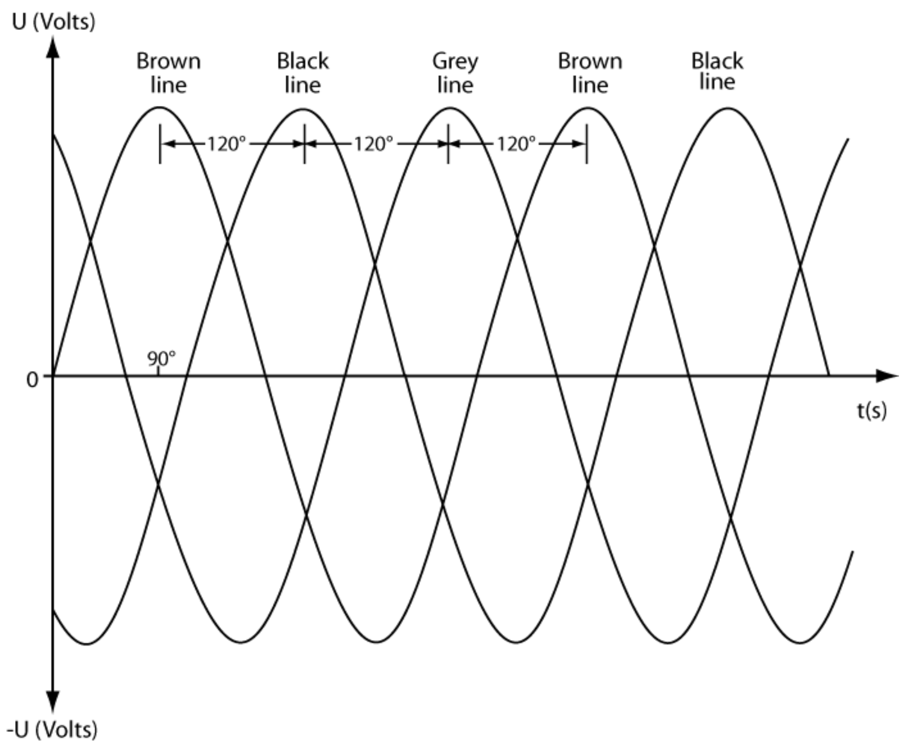
Because there are three windings then there are three induced emfs produced. Because the windings of the generator are set 120º apart the three emfs rise and fall at three different times.

The reason why three phases are generated is to allow industry to reduce the size of the machines that it uses. With a reduction in the size of machines, there will be a corresponding reduction in the size of the cables used to supply the machines. A three-phase system also allows the distributors to reduce their cable sizes and balance the loads attached to their distribution system.



Here we have a diagram showing the general aspects of a generator. You can see the three sets of windings set 120º apart, and the output to the transformer.

The actual supply from a three-phase alternator would look something like the diagram over the page.

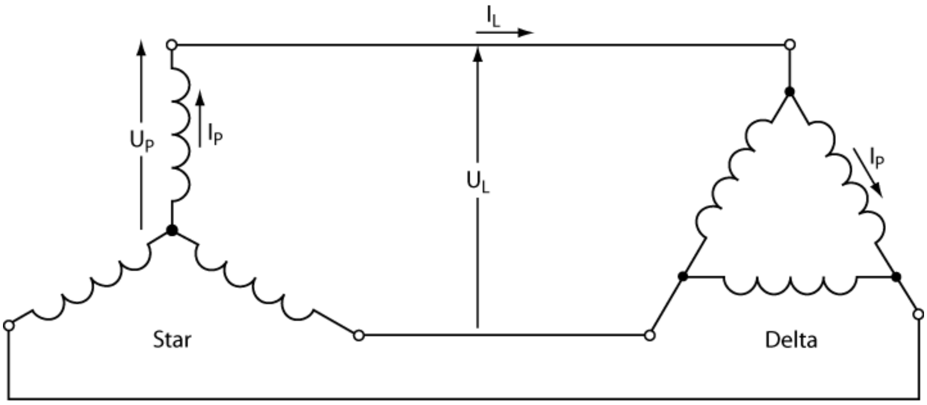


You can see that there are three separate waveforms separated by an even distance. As already stated this distance will be 120˚.

Voltages and currents in a three-phase system

Before we go any further, we'll take a few moments to make sure that you are familiar with the correct labelling. This reduces any possibility of mistakes occurring. More mistakes are made by not recognising the different labels than anything else.

Consider the diagram below, making particular note of the labelling. You will see that there are two ways of connecting a three-phase supply to a load – **star** (where all the ends are connected to a common point) and **delta** (where all the ends are connected to one another).



The subscript 'P' has been used in this diagram to distinguish between the line and phase values of current and voltage

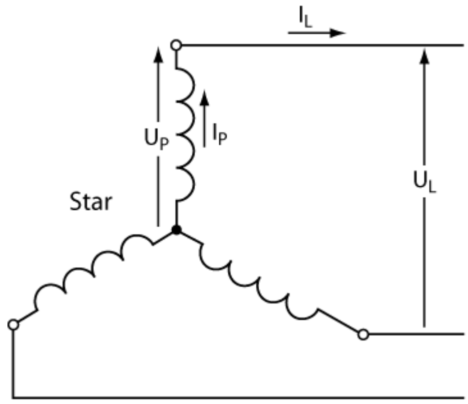
Here we have a star connection attached to a delta connection.

$$\text{Line voltage } U_L = \text{Voltage between any two line conductors}$$
$$\text{Line current } I_L = \text{Current in any line conductor}$$
$$\text{Phase voltage } U_P = \text{Voltage between any line conductor and neutral}$$
$$\text{Phase current } I_P = \text{Current in any phase conductor}$$

Let's initially consider the star connected system. We'll assume that we have a perfectly balanced load, and that whatever is on the brown phase is also on the black and grey phases.

Star-connections

Here is a fully labelled star connected system. We have three line currents; three line voltages; three phase currents and three phase voltages.

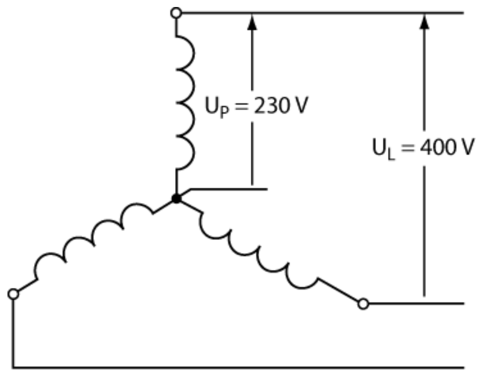


The subscript 'p' has been used in this diagram to distinguish between the line and phase values of current and voltage

What is their relationship to each other? Initially you will notice that the phase current and the line current are the same.

$$I_L = I_P$$

With voltages in the star system, things are a little different. For a start, the neutral provides us with a further option.

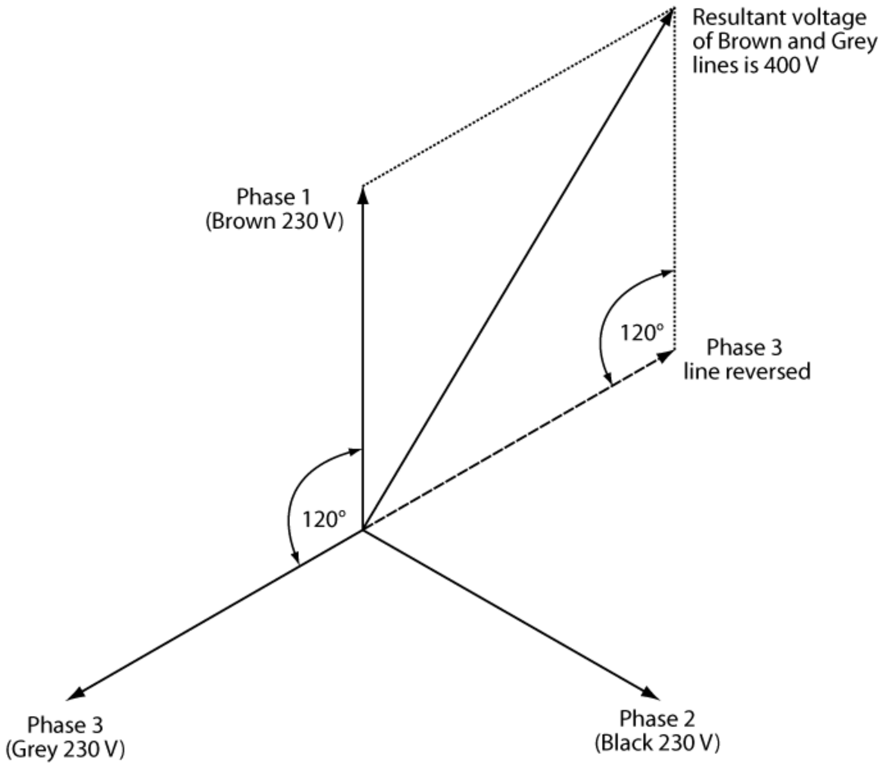


The subscript 'P' has been used in this diagram to distinguish between the line and phase values of current and voltage

You'll notice that each of the voltages is shown as being set 120º apart.

You know from your work with single-phase circuits that electrical quantities can be drawn to scale using a phasor (vector) diagram. If we were now to add any two of these potential differences together, we would get something of an increase.

Have a look at the diagram below.



Here you can see that there hasn't been a doubling of the voltage, but rather a substantial increase. As this is a phasor diagram, there can be no doubling if the voltages are not in line with each other. Now, if each of the potential differences are exactly the same, we can come up with an equation that explains this relationship.

Remember that we are dealing here with a balanced system, such as the windings of a three-phase motor. An unbalanced system is a little more complicated, although we look then more at the currents rather than the voltage.

However for a balanced load the equation states:

$$U_L = \sqrt{3}U_P$$
$$U_P = \frac{U_L}{\sqrt{3}}$$

If you were to measure our phasor diagram above, you should find that the addition of the two phase potential differences is approximately 400 V.

$$U_L = \sqrt{3}U_P$$
$$U_L = \sqrt{3} \times 230 = \underline{\underline{398\text{V}}}$$

So in our general balanced star connected systems we will always find these two relationships.

$$U_L = \sqrt{3}U_P$$
$$I_L = I_P$$

Try filling in this table.

U_L	55	220		24	12	
U_P			600			1500

In a perfectly balanced star-connected system, the current in the neutral wire would be 0A (zero). This is because each current would cancel each other out.

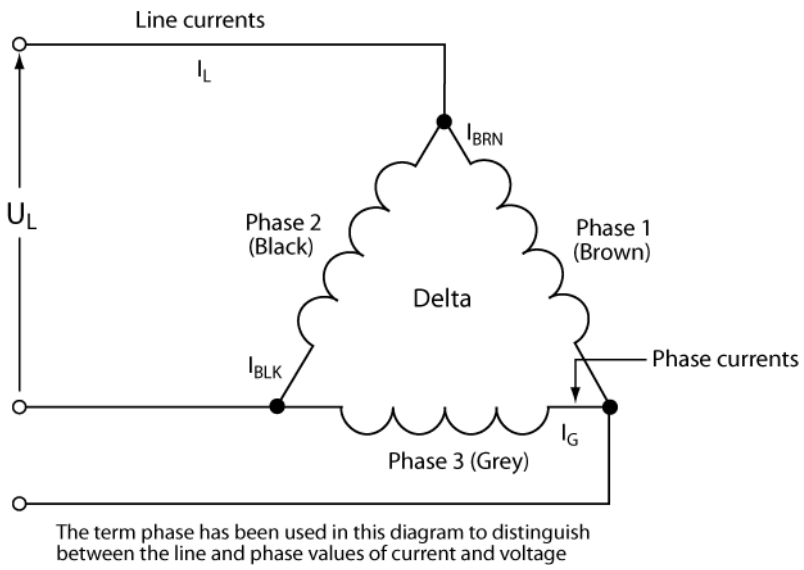
Now we'll look at the delta-connected system.

Delta connections

Again, we can draw the system in the form of a phasor diagram. This time however, our line and phase voltages are the same and our line and phase currents differ.

$$U_L = U_P$$

Therefore, for a 400 V phase voltage, there will be a 400 V line voltage. Notice that this is exactly the opposite of what happens with a star connected system. The diagram below shows a fully labelled delta connected system.

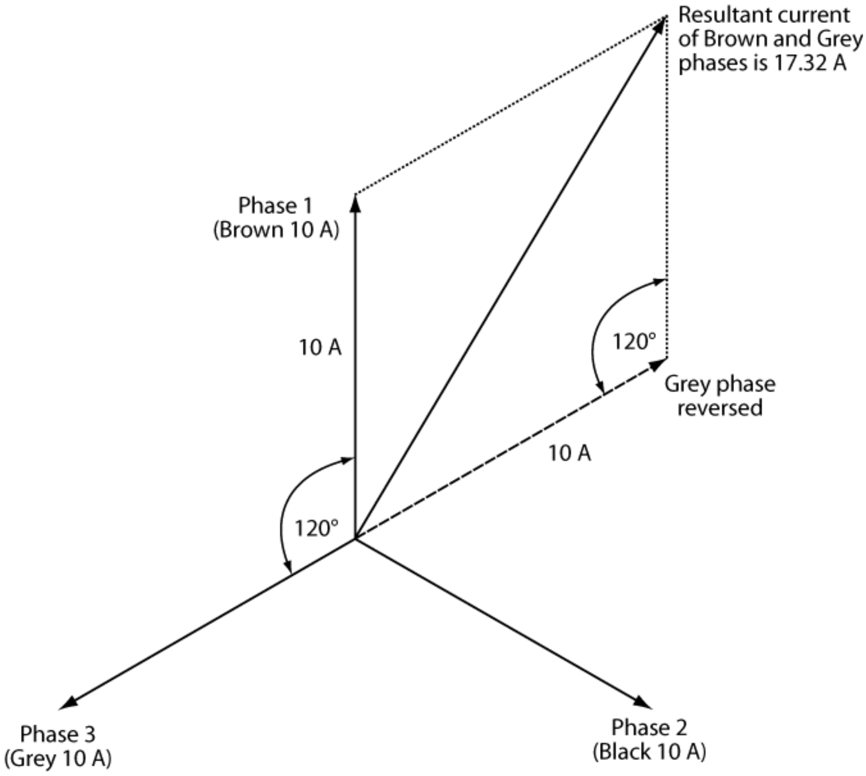


Remember that this time we are dealing with the currents.

$$I_L = \sqrt{3}I_P$$
$$I_P = \frac{I_L}{\sqrt{3}}$$

The voltage remains constant whilst the current varies.

The overall line current is therefore found by subtracting one phase current from another. However, because each of the phase currents are 120º apart, we end up increasing the line current.



The term phase has been used in this diagram to distinguish between the line and phase values of current and voltage

Here we have 10 A flowing in each phase. The overall line current is, similar to the voltage in a star connected system, not quite double each of the phase currents.

As an example, consider a phase current of 10 A in a balanced delta connected system. What would be the line current in any of the conductors feeding that system?

Now try these few examples in the table below.

I_L		100	4.9	210		
I_P	12				60	78

We'll just have a quick review of the equations that have been introduced.

$$\text{Delta} \quad I_L = \sqrt{3} I_P$$
$$U_L = U_P$$
$$\text{Star} \quad I_L = I_P$$
$$U_L = \sqrt{3} U_P$$

Don't forget these two relationships, and the difference between line and phase values.

Exercise 10.

1) How could I increase the current drawn from a supply, simply by changing connections?

2) What advantages are there for industry in using a three-phase supply?

3) Fill in the following table for a star-connected system.

U_L			230	250		11 kV
U_P	55	110			3 750	

4) In a balanced star connected system, what will be the value of the neutral current? Why?

5) What is the relationship between voltage and current in:

i) Star

ii) Delta?

6) Fill in the following table for a delta-connected system.

I_L	100			75		3k
I_P		50	35		185	

7) Three 10 Ω resistors are connected to a balanced three-phase 400 V supply. Calculate the phase and the line current when they are connected in:

i) Delta

ii) Star.

8) Three identical 100 Ω resistors are connected in delta across a 400 V three-phase supply. What is the line current?

11: Power in three phase systems

In this session the student will:

- Reinforce an understanding of power in a.c. circuits and their application in three phase systems.

Power in a three-phase system

We have already seen that in a normal a.c. circuit there are three distinct factors.

- Resistance (R) and depends solely on the length, area and type of material that makes up the load.
- Reactance (X). Reactance is a measure of the inductive or capacitive effect of the circuit, and where resistance opposes current flow the reactance opposes current change. Both are however measured in ohms (Ω).
- Impedance (Z) is a combination of resistance and reactance and is also measured in ohms.

The total supply current takes into account the combination of resistance and reactance and uses impedance as the means of determining it.

$$I_S = \frac{U}{Z}$$

$$I_R = \frac{U}{R}$$

$$I_X = \frac{U}{X}$$

Now these three equations all provide a value of current and they may all be appropriate in any one particular instance.

So which one do we use when we are trying to determine the power consumed?

There are in effect three ways in which we look at power in an a.c. circuit, and we have already looked at all of them; however as a form of revision we will consider them again. The three types of power are called:

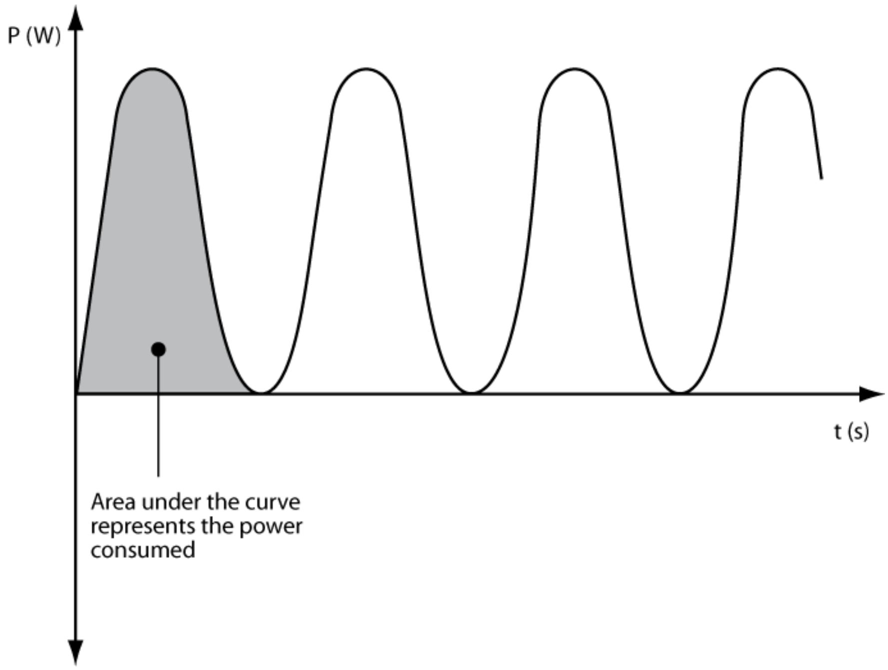
- True Power
- Apparent Power
- Reactive Power.

True Power

True power is a measure of the power used when considering the resistance value only. It is measured in **watts** (**W**) or **kilowatts** (**kW**).

With true power the voltage and current are '**in phase**' with each other. This time however there is no negative part to the cycle. This makes sense as when two negative values are multiplied together then we get a positive value.

The area under the curve of the graph shows the power absorbed by the circuit.



The calculation of true power is the same as for d.c. circuits and is often called the $\left(I^2R\right)$ power or loss. You can think of true power as useful power.

Reactive Power

Reactive power is often called wattless power. It is measured in '**reactive volt-amperes**', '**VAr's**' or '**kVAr's**'.

Reactive power is the measure of the power in terms of either, an inductor or capacitor, and has the symbol '**Q**'.

Here, the supply voltage is 90º out of phase with the current, assuming that we have a pure inductor or capacitor. It doesn't matter now whether we are looking at a purely capacitive or inductive circuit, the consequences are the same.

If the voltage and current are now multiplied together, the power alternates between the positive and negative half cycle. This is because the inductor or capacitor is storing energy on one half-cycle and then returning it to the supply on the next half cycle. The overall effect is that the 'pure' inductor or capacitor uses no power. It is, as its name suggests: wattless!

The pure resistor has $\left(I^2R\right)$ losses and the pure inductor/capacitor has $\left(I^2X\right)$ losses (which are no losses at all). We know however, that we will rarely get a pure resistor or inductor/capacitor.

Apparent Power

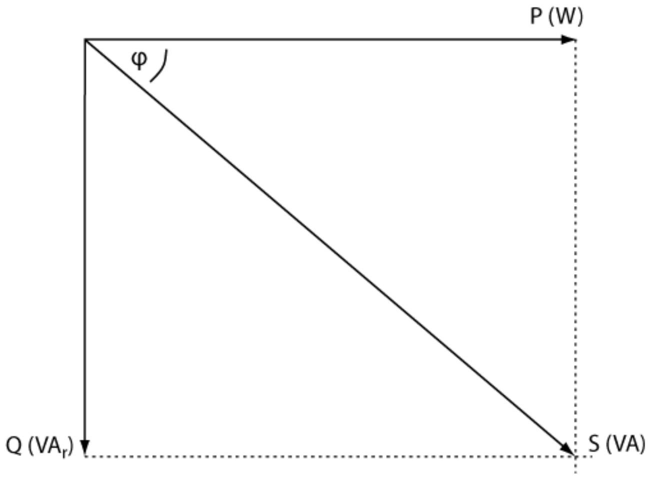
Apparent power is a measure of the combined effects of resistive and reactive power. Because there is no pure resistor, inductor or capacitor, the overall power demand is always going to be a combination of the two.

Apparent power is measured in '**volt-amperes**', '**VA**', or '**kVA**', and has the symbol, '**S**'.

As you will remember, Pythagoras' theorem comes into play here. So:

$$S = UI \quad [\text{VA}]$$
$$P = UI\cos\phi \quad [\text{W}]$$
$$Q = UI\sin\phi \quad [\text{VAr}]$$

These three relationships relate the three sides of the triangle together.



The angle (ϕ) is the angle formed by the apparent power and the true power.

We have already seen that there are a number of relationships between current and voltage within a three-phase system.

$$\begin{aligned} \text{Delta} \quad & I_L = \sqrt{3}I_P \\ & U_L = U_P \\ \text{Star} \quad & U_L = \sqrt{3}U_P \\ & I_L = I_P \end{aligned}$$

If we combine either of these two sets of relationships (for star and delta) to get power, we will have the same overall effect.

$$\begin{aligned} \text{Delta} \quad & I_L = \sqrt{3}I_P \quad \text{and} \quad U_L = U_P \\ & P = 3I_P U_P \quad \text{substitute for } I_L \text{ and } U_L \\ & P = 3 \times \frac{I_L}{\sqrt{3}} \times U_L \\ & \therefore P = \sqrt{3}U_L I_L \\ \text{Star} \quad & I_L = I_P \quad \text{and} \quad U_L = \sqrt{3}U_P \\ & P = 3I_P U_P \quad \text{substitute for } I_L \text{ and } U_L \\ & P = 3 \times \frac{U_L}{\sqrt{3}} \times I_L \\ & \therefore P = \sqrt{3}U_L I_L \end{aligned}$$

This is not to say that the two values of power will be the same in any one system. What I am saying is that the equation used to determine each is the same.

In the delta system, the voltage will be greater than the star system and so the power will be greater in a delta system for any given set of conditions.

There is one more factor that we also need to consider, and that is power factor.

In the same way as a single-phase supply, a three-phase supply has a power factor that needs to be considered. This is in effect no more complex than what you are used to in a single-phase system.

The two equations below show how power factor is taken into account when considering a three-phase system.

$$P = \sqrt{3}U_L I_L \cos\phi$$

$$\text{transposing} \quad I_L = \frac{P}{\sqrt{3}U_L \cos\phi}$$

$$\text{transposing again} \quad \cos\phi = \frac{P}{\sqrt{3}U_L I_L}$$

I have transposed the equation so that you can see how the line current (the current in the conductor) can be determined. This is particularly useful when looking at cable calculations.

The other factor that may sometimes have to be considered is efficiency. When considered as a per unit value it is straightforward to combine all these factors at one go.

$$P = \sqrt{3}U_L I_L \cos\phi\eta$$

$$\text{transposing} \quad I_L = \frac{P}{\sqrt{3}U_L \cos\phi\eta}$$

$$\text{transposing again} \quad \cos\phi = \frac{P}{\sqrt{3}U_L I_L \eta}$$

Here the efficiency has just become another factor to consider along with the power factor.

1). Three 25 Ω resistors are connected in star and then delta across a 400 V three-phase supply. Determine:

i) the line and phase currents
ii) the line and phase voltages
iii) the power.

$$U_L = \sqrt{3}U_P \quad \text{transposing}$$
$$U_P = \frac{U_L}{\sqrt{3}}$$
$$U_P = \frac{400}{\sqrt{3}} = \underline{\underline{231\text{V}}}$$
$$I_P = \frac{U_P}{R} = \frac{231}{25} = \underline{\underline{9.24\text{A}}}$$
$$I_L = I_P = 9.24\text{A}$$
$$P = \sqrt{3}U_L I_L$$
$$P = \sqrt{3} \times 400 \times 9.24 = \underline{\underline{6401\text{W}}} = \underline{\underline{6.4\text{kW}}}$$

or you can do it this way

$$P = 3I_P U_P$$
$$P = 3 \times 9.24 \times 231 = \underline{\underline{6401\text{W}}} = \underline{\underline{6.4\text{kW}}}$$

These are the values for the star connected system. Be very careful to pick the right values. The delta-connected system is shown below.

$$U_L = U_P = 400\text{V}$$
$$I_P = \frac{U_P}{R} = \frac{400}{25} = \underline{\underline{16\text{A}}}$$
$$I_L = \sqrt{3}I_P$$
$$I_L = \sqrt{3} \times 16 = \underline{\underline{27.7\text{A}}}$$
$$P = \sqrt{3}U_L I_L$$
$$P = \sqrt{3} \times 400 \times 27.7 = \underline{\underline{19191\text{W}}} = \underline{\underline{19.2\text{kW}}}$$

or you can do it this way

$$P = 3I_P U_P$$
$$P = 3 \times 16 \times 400 = \underline{\underline{19191\text{W}}} = \underline{\underline{19.2\text{kW}}}$$

You will notice the large increase in the power consumed. It is three times that for a star connected system.

2). Three resistors of value 8 Ω are connected across a 110 V three-phase supply. Determine all values for both a star and delta connected system.

We'll look at the star system first.

$$U_L = \sqrt{3}U_P \quad \text{transposing}$$
$$U_P = \frac{U_L}{\sqrt{3}}$$
$$U_P = \frac{110}{\sqrt{3}} = \underline{\underline{63.51\text{V}}}$$
$$I_P = \frac{U_P}{R} = \frac{63.51}{8} = \underline{\underline{7.94\text{A}}}$$
$$I_L = I_P = 7.94\text{A}$$
$$P = \sqrt{3}U_L I_L$$
$$P = \sqrt{3} \times 110 \times 7.94 = \underline{\underline{1513\text{W}}} = \underline{\underline{1.5\text{kW}}}$$

Now for the delta system.

$$U_L = U_P = 110\text{V}$$
$$I_P = \frac{U_P}{R} = \frac{110}{8} = \underline{\underline{13.75\text{A}}}$$
$$I_L = \sqrt{3}I_P$$
$$I_L = \sqrt{3} \times 13.75 = \underline{\underline{23.75\text{A}}}$$
$$P = \sqrt{3}U_L I_L$$
$$P = \sqrt{3} \times 110 \times 23.75 = \underline{\underline{4538\text{W}}} = \underline{\underline{4.53\text{kW}}}$$

You should be able to see the process involved here by now.

3). A 400 V three-phase motor is connected in star and delivers a load of 3 kW. It has an efficiency of 0.75 p.u. and a power factor (pf) of 0.75. Calculate the line current and the apparent power.

$$\eta = \frac{\text{output power}}{\text{input power}} \quad \text{transposing}$$

$$\text{Input power} = \frac{\text{output power}}{\eta}$$

$$\text{Input power} = \frac{3000}{0.75} = \underline{\underline{4000\text{W}}}$$

$$P = \sqrt{3}U_L I_L \quad \text{transpose}$$

$$I_L = \frac{P}{\sqrt{3}U_L \cos\phi}$$

$$I_L = \frac{4000}{\sqrt{3} \times 400 \times 0.75} = \underline{\underline{7.7\text{A}}}$$

$$S = \sqrt{3}U_L I_L$$

$$S = \sqrt{3} \times 400 \times 7.7$$

$$S = \underline{\underline{5335\text{VA}}} = \underline{\underline{5.34\text{kVA}}}$$

Above you can see the difference between the apparent power at the bottom (5.34 kVA) and the true power of 4 kW.

You would be paying for all the energy used (the apparent power) and not just the true power, which is the useful power necessary for the equipment to function.

4). A balanced star-connected three-phase load consists of a resistor of 5 Ω and an inductor of 31.8 mH connected in each phase. If the voltage is 400 V and the frequency is 50 Hz, determine the following:

a) Inductive reactance
b) Impedance
c) Power factor
d) Line current
e) Apparent power
f) True power.

There is a lot of ground to cover here, but if you deal with each part of the problem in turn, all you have is a series of small problems. Don't consider anything else until you have the first bit of information.

$$X_L = 2\pi fL$$
$$X_L = 2\times\pi\times 50\times 0.0318 = \underline{\underline{10\Omega}}$$
$$Z = \sqrt{R^2 + X^2}$$
$$Z = \sqrt{5^2 + 10^2} = \sqrt{125} = \underline{\underline{11.2\Omega}}$$
$$\cos\phi = \frac{R}{Z} = \frac{5}{11.2} = 0.447$$
$$\phi = \cos^{-1} 0.447 = \underline{\underline{64.43^\circ}}$$
$$U_P = \frac{U_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \underline{\underline{231\text{V}}}$$
$$I_L = I_P = \frac{U_P}{Z} = \frac{231}{11.2} = \underline{\underline{20.63\text{A}}}$$
$$S = \sqrt{3}U_L I_L$$
$$S = \sqrt{3}\times 400\times 20.63 = \underline{\underline{14293\text{VA}}} = \underline{\underline{14.3\text{kVA}}}$$
$$P = \sqrt{3}U_L I_L\cos\phi$$
$$P = \sqrt{3}\times 400\times 20.63\times 0.447 = \underline{\underline{6389\text{W}}} = \underline{\underline{6.4\text{kW}}}$$

Remember that the phase and line current in a star system are the same.

This is our apparent power. Remember this is the actual power being paid for, not the useful power.

Because our power factor is so poor in this instance, you can see that the true power is less than half the value of the apparent power. Remember that it is best if the true and apparent powers are close in value to each other.

You should see in this problem that the length of the question does not make it difficult. If you can reduce the problem into small chunks then things are a lot easier.

Try completing the tables below. The first table is a star-connected system. Determine all unknown values.

Efficiency	0.9	0.8	1.0	0.75	0.65	0.95	0.88
True Power							
Apparent Power							
Line Current	10	15	25	5	14	125	220
Phase Current							
Line Voltage		400	110			415	240
Phase Voltage	230			50	220		
Power Factor	0.7	1	0.9	0.8	0.55	0.75	0.95

The table below is a delta-connected system. Determine all the unknown values.

Efficiency	0.9	0.8	1.0	0.75	0.65	0.95	0.88
True Power							
Apparent Power							
Line Current	10	15	25	5	14	125	220
Phase Current							
Line Voltage		400	110			415	240
Phase Voltage	230			50	220		
Power Factor	0.7	1	0.9	0.8	0.55	0.75	0.95

We have again covered a fair degree of ground, and it would be worthwhile for us to have a brief summary of exactly what we have done.

	Phase Current	**Line Current**	**Phase Voltage**	**Line Voltage**
Star	$I_P = I_L$	$I_L = I_P$	$U_P = \frac{U_L}{\sqrt{3}}$	$U_L = \sqrt{3}U_P$
Delta	$I_P = \frac{I_L}{\sqrt{3}}$	$I_L = \sqrt{3}I_P$	$U_P = U_L$	$U_L = U_P$

Apparent power: $S = \sqrt{3}U_L I_L$

True power: $P = \sqrt{3}UI\cos\phi$

Reactive power: $Q = \sqrt{3}U_L I_L \sin\phi$

Exercise 11.

1) A 400 V, 25 kW three-phase induction motor has an efficiency of 85% and a power factor of 0.83. What will be the input line and phase currents:
   i) If the motor is connected in delta
   ii) If the motor is connected in star?

2) Draw a phasor diagram showing a reactive power of 50 KVAr and a true power of 125 kW. Determine:
   i) Apparent power
   ii) Power factor.

3) Draw a phasor diagram showing true power of 75 kW and apparent power of 100 kVA. Determine:
   i) Reactive power
   ii) Power factor.

4) A commercial installation draws current of 112 A per phase on a three-phase 400 V supply. The assessed power factor is 0.92. Determine:
   i) True power
   ii) Apparent power
   iii) Reactive power.

5) If we were paying for the electricity, what type of power would we want to be charged for, and what type of power are we actually charged for?

12: Neutral currents in three phase systems

In this session the student will:

- Gain an understanding of how to determine the neutral current in a three phase four wire supply.

In the previous sessions we have considered the nature of three phase star and delta connected loads and have seen how true, reactive and apparent power can be calculated. In this session we will consider the particular case of where we are no longer dealing with balanced loads but rather with unbalanced ones, and in these loads we have to consider what happens to the current flowing in the neutral conductor.

We have considered, up to this point, a perfectly balanced system. What happens when things are not so balanced?

This takes a little bit more thought so follow carefully. Consider this situation.

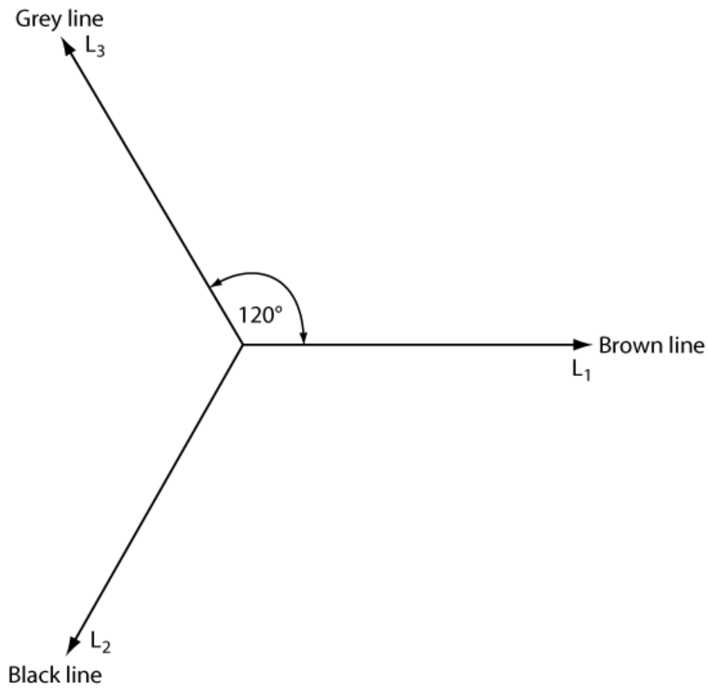
1). A star connected system has three loads connected to the three phases.

- On the brown phase, there is a load totalling 50 A at unity power factor.
- On the black phase, there is a load totalling 75 A at unity power factor.
- On the grey phase, there is a load totalling 65 A at unity power factor.

Calculate the neutral current.

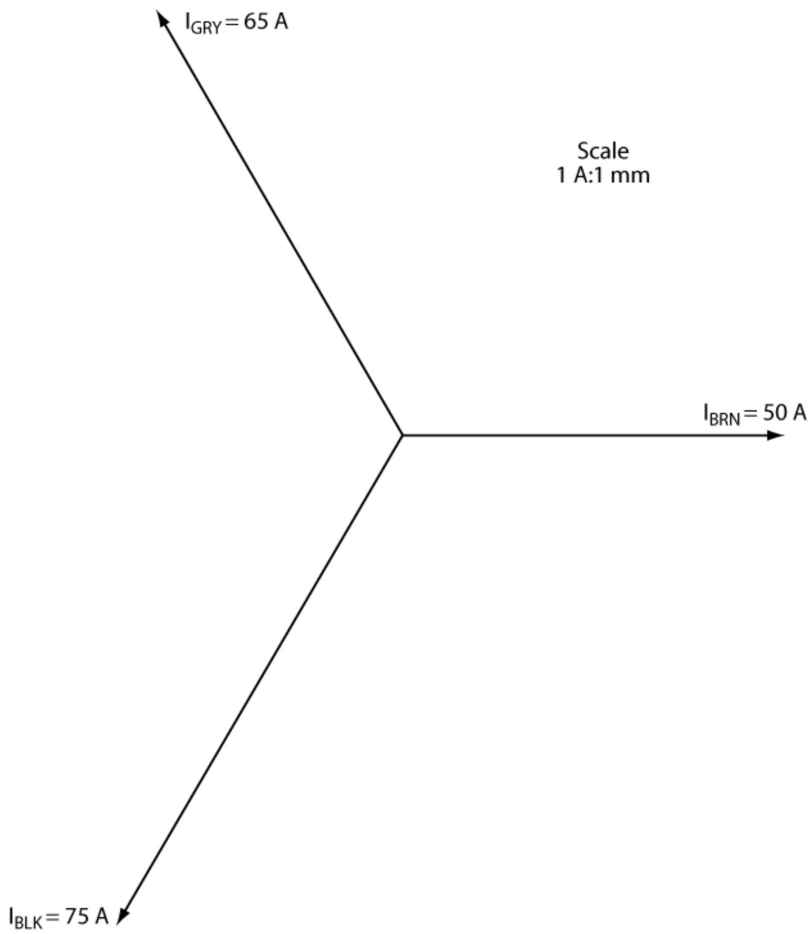
Now normally we might look at this and think that because we have a star connected system and all the currents are the same, then we must have a balanced system and therefore the neutral current must be zero (0 A). However, as you look at the phasor diagram, you will see that the changes in the phase angles (caused by the differing power factors) will lead to an unbalanced system and the production of an actual neutral current.

Follow the diagrams through carefully as we construct a phasor diagram for an unbalanced system.



In the above diagram we have just drawn in the reference lines for each of the three phases. We are drawing them 120º apart.

Now we are going to draw in the loads with reference to their respective reference lines.



You can see that the brown phase line of 50 A drawn to scale has been superimposed on top of our reference. This is because it is '**in phase**' with it.

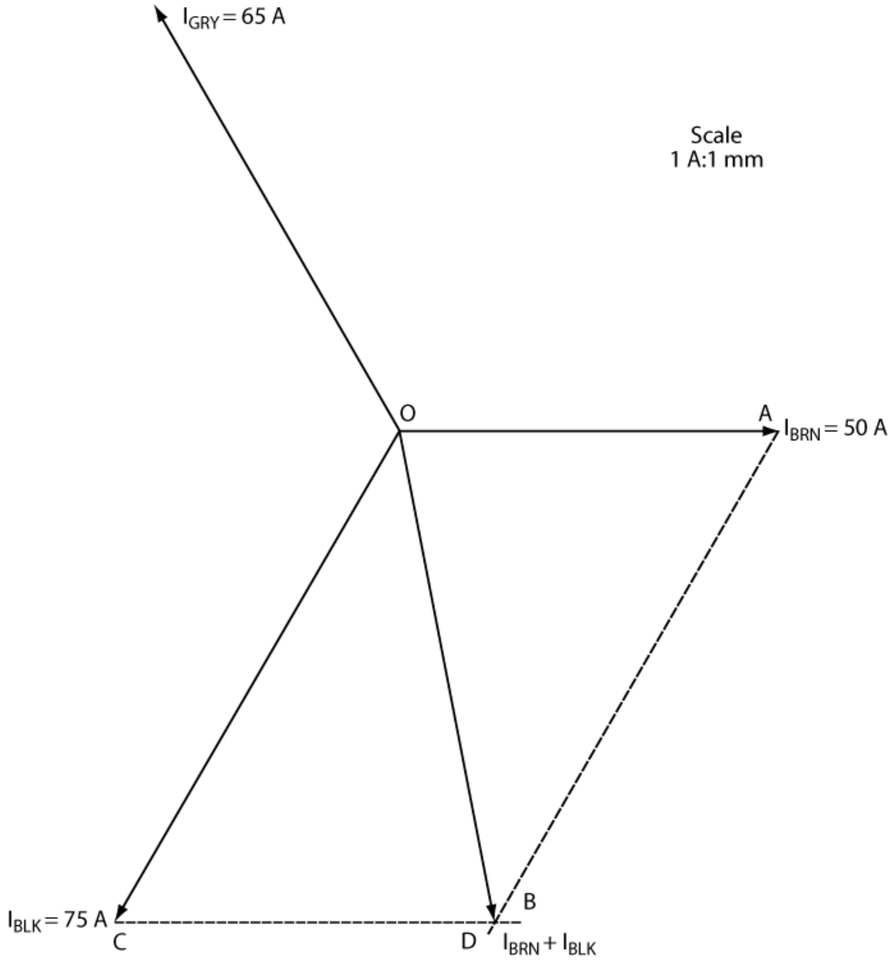
The black phase current line of 75 A is now drawn on top of the black phase reference line, and the grey phase line of 65 A is drawn on top of the grey phase reference line. Now we can get rid of the reference lines.

Remember that we are dealing with each of these loads as if they are just three single-phase loads at the moment. It just so happens that they are all being drawn on the same diagram. The actual angle is related to the power factor, as you already know.

We can now start to add these lines together as the combined current of all of the three phases make up the neutral current.

$$I_N = I_{Br} + I_{Bl} + I_G$$

Have a look at the diagram over the page.

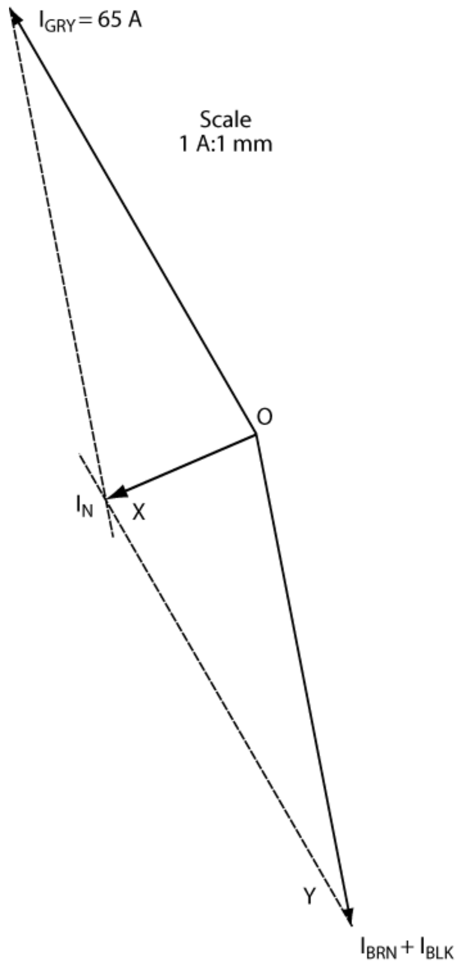


In the diagram above you can see that a line (**AB**), parallel with the black phase, has been drawn from the end of the brown phase.

You will also see that a line (**CD**) has been drawn, parallel with the brown phase, from the end of the black phase.

The two lines cross. If we draw a line from the centre (star-point) of our diagram to the position where they cross, we will have added the brown to the black phase. We now have to add this value to the grey phase.

Again, look at the diagram below.



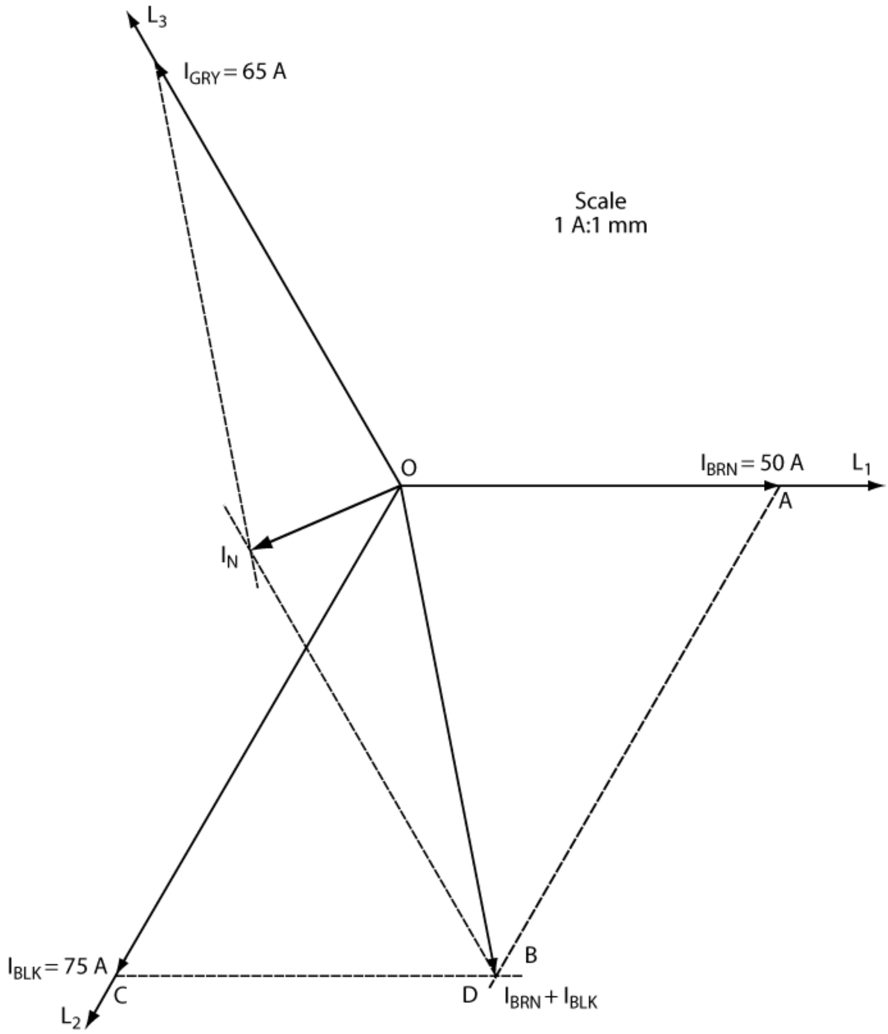
Here we are merely repeating what we did on the previous page.

A line, parallel with the grey phase, is drawn from the end of our newly added value for the brown and black phases (**XY**). A second line, parallel with the brown/black phase addition is drawn from the end of the grey phase.

The two lines again should cross. If we draw a line from the centre (star-point) of our diagram to the position where they cross, we will have added the brown/black phase to the grey phase.

The length and direction of the line provide us with the value of the neutral current. This is the actual current that will flow in a neutral conductor, whilst the direction on the diagram tells us the phase angle compared to the other three phases.

If you measure your diagrams very carefully, you should find that in this instance the neutral current is approximately 22 A and is leading the grey phase by approximately 84º. Have a look at the complete phasor diagram below.



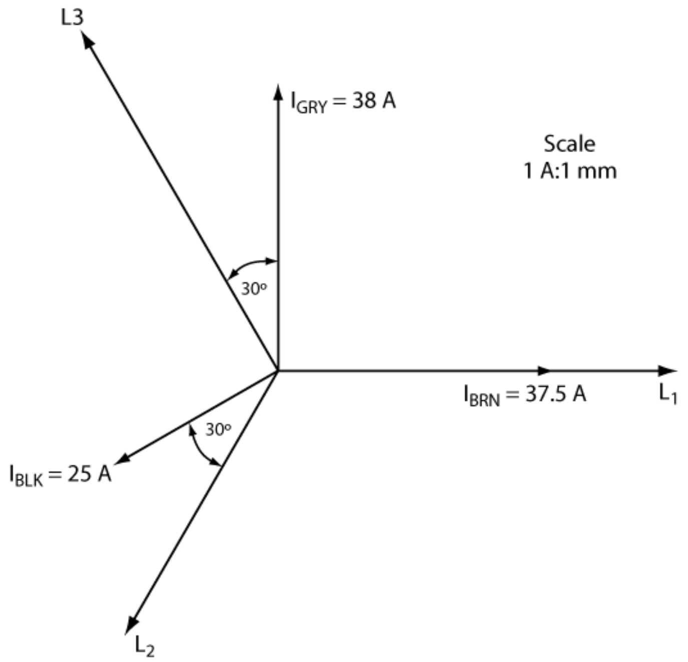
Try another example.

2). The line currents taken by a three-phase, four-wire installation are as follows:

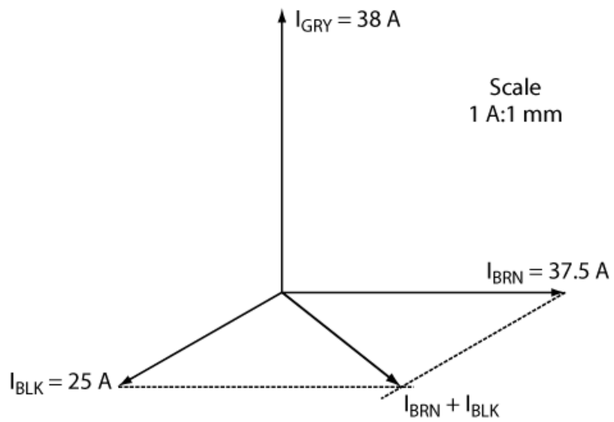
- Brown phase 35.7 A in phase
- Black phase 25 A lagging by 30º
- Grey phase 38 A lagging by 30º.

Determine the neutral current and the direction in which it operates.

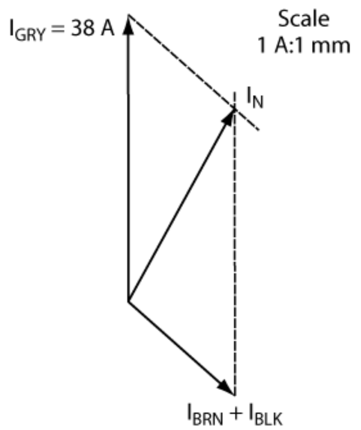
In this instance we have to be a little more careful where we position our lines compared to the reference values. Remember to draw the reference lines in first and then superimpose the actual current values on top of it. The phasor diagram is shown below.



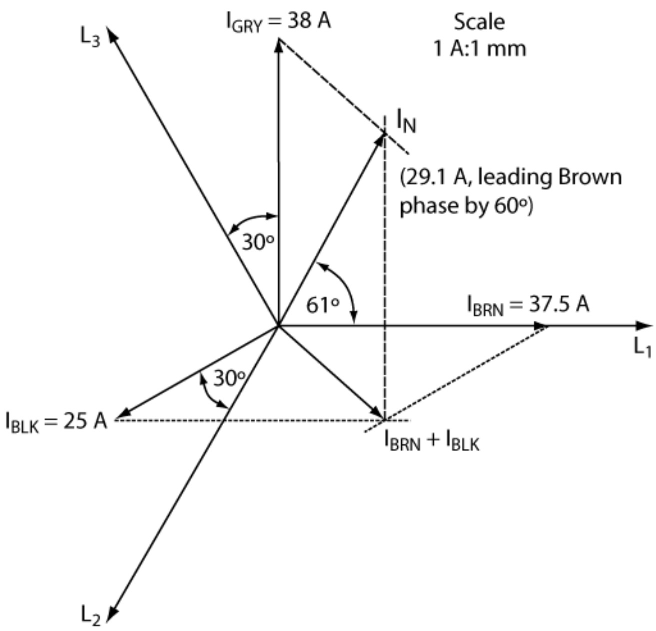
Now we have the values set on our phasor diagram let's consider the brown and black phases as we did with the first example. Draw parallel lines from the end of the brown phase and from the end of the black phase. Remember, that where they meet we draw a line.



Once we have this part of the diagram we then deal with the now combined value of brown and black and the grey phase.

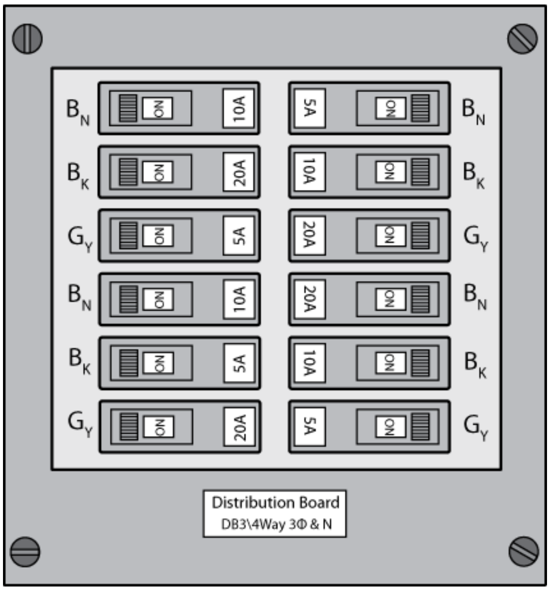


The trick is to make sure that you know what the angles are that you are dealing with. Get this bit wrong however, and you will make a 'pigs ear' of the answer!



An understanding of this principle has a very practical use.

Assume that you are looking at a 4-way TP&N (Triple Pole and Neutral) distribution board. This means that this board has twelve single pole outlets for protective devices or four triple pole devices.



If each of these single pole devices is supplying a load, it doesn't matter how good we are at trying to balance the varying loads, based on the current ratings of the protective devices, we know that as things get switched on and off there will be a variation in the neutral current.

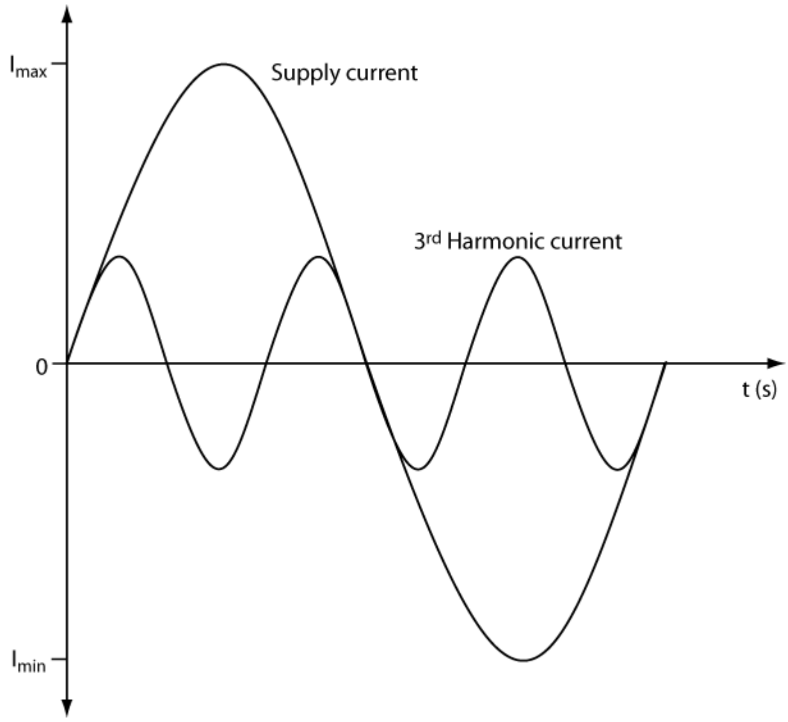
This doesn't take away the responsibility that we have to try and balance our loads, but it does mean that not only should we take into account the currents that a particular load may demand, but also the power factor of that particular load and also how often it will be turned on.

This is why of course that BS 7671 deals with neutral conductors in a variety of circumstances. Consider Section 524 of BS 7671, and in particular, Regulation 524.2.3. Explain why it is permitted to have a reduced neutral.

You are now faced with a much clearer need to consider the **'inequality of phase loading**', the '**inequality of power factor in each phase**' and '**any harmonic currents**'. You must be familiar with the process. It is not acceptable merely to choose a neutral conductor that is the same size as the line conductor if the neutral current may end up being larger than the phase currents.

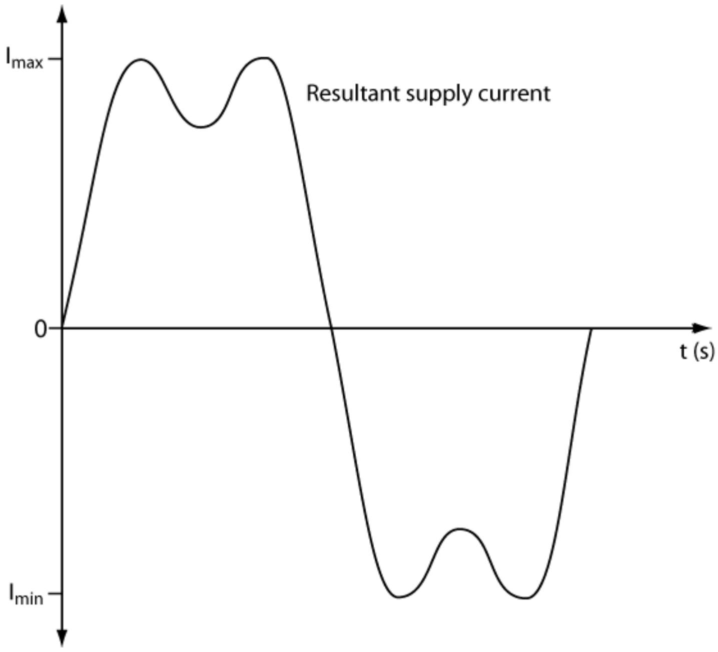
An additional problem arises when there are very heavy inductive loads. In some loads there are additional factors that need to be considered which are called **harmonic currents**.

In any inductive circuit there are additional currents created. We are already aware of them. We call one additional current the induced current. However, with harmonic currents there is an added effect. Consider the diagram below.



Here the supply current is shown, but superimposed is an additional sine wave. This additional sine wave has three cycles for every one of the main supply current. This is where we get the label of third harmonic from. This is a way of modelling the effect of having certain inductive circuits switched like, chokes in fluorescent fittings, inverters etc.

If we add up the overall effect, there is an increase in the overall current and, a messing up of our nice main sine wave. Have a look at the diagram below. This is not exactly what is happening however, but rather a mathematical technique to model the effect.



Here you can see that the total current has increased. For an inductive load therefore, we need to consider, not only the current that we can calculate, but we also need to consider the additional effects. For some very heavy inductive loads we would need to consider the combined effects of the third and other harmonics, although this is rare.

Exercise 12.

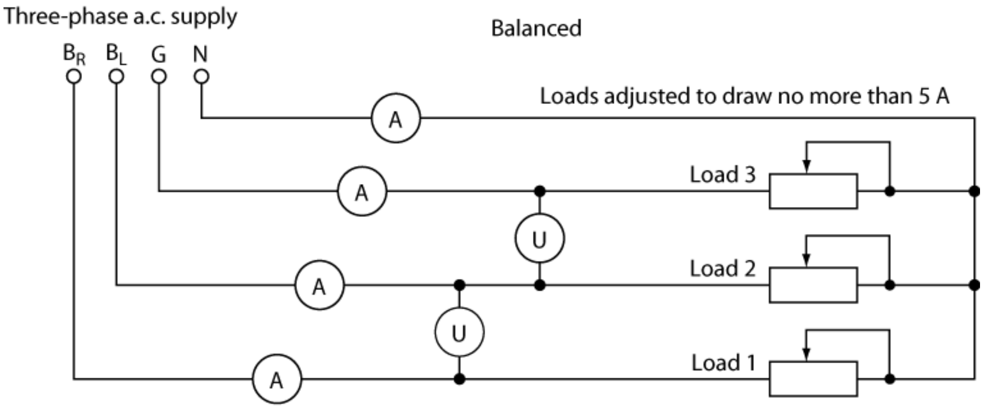
1) In a balanced star connected system, what will be the value of the neutral current? Why?

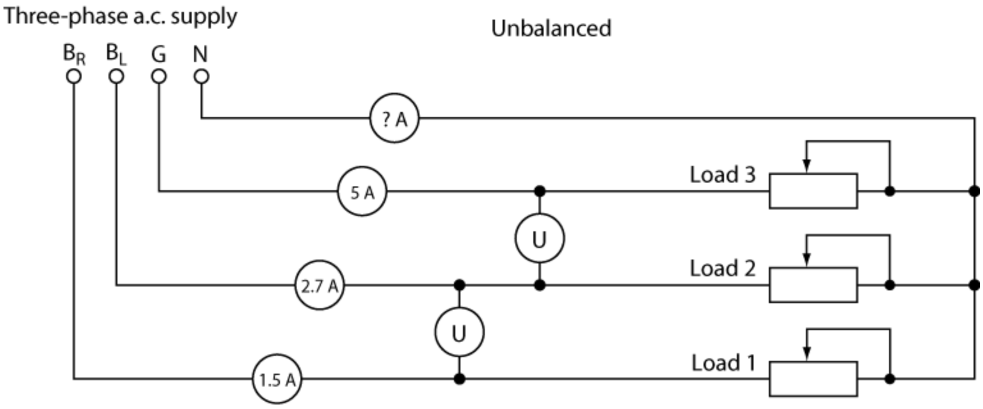
2) Harmonic currents can increase the current drawn by a supply. What types of loads increase this effect?

3) Determine the value of the neutral current in a three-phase, four-wire system when the following currents are drawn:

Brown phase	95 A	25.8º lag
Black phase	105 A	45.6º lag
Grey phase	70 A	25.8º lead

4) Set up an experiment to measure the neutral current in a balanced and an unbalanced three-phase load.





Measure the values of current in each phase.

Measure the current in the neutral conductor.

If you have used pure resistors you should be able to check by drawing a phasor diagram whether the theory is true.

End of unit questions

1). What are the angles for the following cosine values:

i) 0.63

ii) 0.89

iii) 0.97

iv) 0.55. 4

2). What would be the rms. values for the following peak values:

i) 700 V

ii) 35 A

iii) 750 V

iv) 150 V 4

3). A resistor of 75 Ω is connected to a 200 V supply. What will be the current drawn from the supply? 2

4). A 100 mH inductor is connected to a 24 V 250 Hz supply. What current will be drawn from the supply? 2

5). A 100 nF capacitor is connected across a 12 V 5 kHz a.c. supply. What current will be drawn? 2

6). A circuit contains an inductor of inductance 47 mH and resistance 56 Ω. If the supply is 80 V, 50 Hz, determine:

i) Reactance 1

ii) Impedance 1

iii) Current 1

iv) Phase angle 1

v) Draw a phasor diagram. 2

7). What changes would take place to Q.6 if the frequency were to double? 6

8). A 20 µF capacitor is connected in series with a 65 Ω resistor across a 220 V, 50 Hz supply. Determine:

i)	Reactance	1
ii)	Impedance	1
iii)	Current	1
iv)	Phase angle	1
v)	Draw a phasor diagram	2

9). What changes would take place to Q.8 if the frequency were to double? 6

10). An inductor of inductance 100 mH and resistance 70 Ω is connected in series with a 180 µF capacitor across a 230 V, 50 Hz supply. Determine:

i)	Reactances	2
ii)	Impedance	2
iii)	Current	1
iv)	Phase angle	1
v)	Draw a phasor diagram	3

11). What changes would take place to Q.10 if the frequency were to double? 9

12). Describe the principle of resonance in a series circuit. 3

13). An inductor of inductance 35 mH and resistance 31 Ω is connected in series with a capacitor of capacitance 500 nF across a 6 V signal generator.

i)	Determine resonance frequency	3
ii)	Plot a graph (frequency being the horizontal axis) of the various elements from 700 Hz to 1 500 Hz in 100 Hz steps.	4

14). A coil has a reactance of 50 Ω and resistance 61 Ω. A capacitor of reactance 35 Ω is connected in parallel with it. The supply is 230 V, 50 Hz. Draw a phasor diagram showing the relationship between the supply voltage and the current flow in each leg. 4

15). Using the values from Q.10 determine:

- i) True power 1
- ii) Apparent power 1
- iii) Reactive power 1
- iv) Draw a power triangle showing the different relationships. 2

16). State two ways in which the power factor of an installation can be improved. 2

17). Give three reasons why the power factor of an installation should be improved. 3

18). What happens when the power in a three-phase system is unbalanced? 2

19). The following loads are connected to a three-phase system.

- Brown phase draws 110 A of current
- Black phase draws 78 A of current
- Grey phase draws 90 A of current.

What will be the neutral current assuming each load is in phase? 6

Total marks 88

